MMDSC-2.1

Sl.No. :

Total No. of Pages : 3

Second Semester M.Sc. Degree Examination, May - 2026
(SLM Scheme) (CBCS)
MATHEMATICS
Algebra - II

Time : 3 Hours **Max. Marks : 80**

Instructions : 1) Answer any Five full questions.
2) All questions carry equal marks.

Q1) a) Let M be an A - module. Prove that the following are equivalent :

i) M is a Northerian A-module

ii) Every non Empty class of submodules of M has a maximal Element.

iii) Every submodule of M is finitely generated.

b) Prove that spec A of a local ring A is quasi compact.

c) Suppose I is an ideal of A. Then prove that I^e is also an ideal.

[8 + 5 + 3]

Q2) a) Prove that if spec A is connected then 0 & 1 are the only idempotents of A.

b) Describe the ring of fractions $S^{-1}A$ of A with respect to a multiplicatively closed subset S of A. Show that there is a bijective correspondence between the prime ideals of $S^{-1}A$ and the prime ideals of A which do not meet S.

[9 + 7]

Q3) a) Show that an Artinian integral domain is a field. Deduce that any Artinian ring is a semi-local ring.

b) Prove that in an Artinian ring the nilradical is nilpotent.

[8 + 8]

Q4) a) Show that M is a finitely generated A-module if and only if M is isomorphic to a quotient of a free A-module.

b) If $I_1, I_2, \ldots\ldots, I_n$ are ideals of a ring A which are pairwise Co-prime then show that $A \Big/ \prod_{k=1}^{n} I_k \cong \prod_{k=1}^{n} A \Big/ I_k$

[8 + 8]

Q5) a) Let K/F be field Extension $\alpha, \beta \in K$ be algebraic over K. If $\deg_F^{(\alpha)} = m$ and $\deg_F^{(\beta)} = n$, then prove that $[F[\alpha, \beta], F] \leq mn$.

b) Let K/F be a field extension and $\alpha \in K$ be algebraic over F. Then prove that $F(\alpha) = F[\alpha]$.

c) Find the degree of Extension of $Q\left(\sqrt{a}, i\right)$ over Q.

[6 + 5 + 5]

Q6) a) Find the splitting field of the polynomial $x^4 + x^2 + 1$ over Q and also degree of extension of the splitting field over Q.

b) Let K be an Extension of field of characteristics 0 and $\alpha \in K$ be algebraic over F. If β is a conjugate of α, then prove that the minimal polynomial of β is some as the minimal of α.

c) Find the degree of Extension of the splitting field of irreducible polynomial $x^3 + px + 2$ over Q.

[5 + 5 + 6]

Q7) a) Let K be a finite field of characteristic P. Then prove that K has a subfield isomorphic to $\mathbb{F}_p$ or $\mathbb{Z}_p = \frac{\mathbb{Z}}{p\mathbb{Z}}$.

b) Prove that for any P and positive integer n, there Exist finite field of order p^n.

[8 + 8]

Q8) a) Prove that Every Quadratic Extension is a Galois extension.

b) State and prove fixed field theorem.

c) Prove that K is a normal Extension of F if and only if K is the splitting field of some polynomial over F.

[5 + 6 + 5]

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