MMDSC-2.1

Sl.No. :

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Second Semester M.Sc. Degree Examination, October - 2025
(SLM Scheme) (CBCS)
MATHEMATICS
Algebra - II

Time : 3 Hours **Max. Marks : 80**

Answer any five full questions. All questions Carry Equal marks.

Q1) a) Define the Nilradical N(A). Show that the nilradical N(A) is the intersection of all prime ideals of A.

b) Define a local ring. Prove that a ring is a local ring if and only if for each $x \in A$ Either x or 1-x is a unit in A. Deduce that the only idempotents in a local ring are 0 & 1.

[8 + 8]

Q2) a) Define the prime spectrum spec A of a ring. Prove that Spec A is disconnected if and only if A contains an idempotent $\neq 0,1$.

b) Prove that Spec A is Compact.

[8 + 8]

Q3) a) State and prove Nakayama's Lemma.

b) State and prove Sehur's Lemma.

[8 + 8]

Q4) a) Define a module of finite length. Show that a module is of finite length if and only if it is both Noetherian and Artinian.

b) Prove that the nilradical of an Artinian ring is nilpotent.

[8 + 8]

Q5) a) Let K be an finite Extension of field F & L be a finite Extension of K. Then prove that L is a finite extension of F and [L : F] = [L : K] [K : F].

b) Find the degree of Extension of $Q\left[\sqrt{2},\sqrt{3}\right]$ over Q.

[8 + 8]

Q6) a) State and Prove Kronecker lemma.

b) Find the splitting field and the degree of Extension of the splitting field of $f(x)$ over Q.

i) $f(x) = x^6 + x^3 + 1$

ii) $x^p - 2$ where p is a prime

[8 + 8]

Q7) a) Prove that set of all constructible numbers form a subfield of $\mathbb{R}$.

b) For any prime p and positive integer n, then prove that there exist field of order p^n.

[8 + 8]

Q8) a) Let K/F be a finite separable Extension. Then prove that $K = F(\alpha)$ for some $\alpha \in K$.

b) State and prove primitive Element theorem.

[8 + 8]

