MMDSC-2.2

Sl. No.

II Semester M.Sc. Degree Examination, December - 2023
MATHEMATICS
Real Analysis-II
(CBCS Scheme)

Time : 3 Hours **Maximum Marks : 80**

Answer any Five full questions.

1. a) If P* is a refinement P, then prove the following.

i) $L(p,f,\alpha) \leq L(p^*,f,\alpha)$

ii) $U(p^*,f,\alpha) \leq U(p,f,\alpha)$

b) If $f \in R(\alpha)$ on [a,b] $f \in R(\alpha_2)$ on [a,b] and $f \in R(\alpha_1+\alpha_2)$ on [a,b] then prove that

$$\int_a^b f\,d(\alpha_1+\alpha_2) = \int_a^b f\,d\alpha_1 + \int_a^b f\,d\alpha_2$$ **(8+8)**

2. a) Let $f \in R(\alpha)$ on [a,b] and let $g(x) = \int_a^x f(t)dt$ for all $x \in$ [a, t]. Then g (x) is continuous on [a,b]. further if f is continous at $x_0 \in [a,b]$ then g is differentiable at x_0 and show that $g^1(x_0) = f(x_0)$.

b) If γ^1 is continuous function on [a, b] then γ is rectifiable then P. T. $\wedge(\gamma) = \int_a^b \|\gamma^1(t)\|\,dt$

c) Evaluate $\int_0^3 x\,d([x]-x)$ **(7+7+2)**

[P.T.O.

3. a) Define uniform convergence, State and prove Cauchy criterion for uniform covergence.

b) Let α be monotonically increasing on [a,b] suppose $f_n \in \mathbb{R}(\alpha)$ on [a,b] for n=1, 2, 3, ..., and suppose $f_n \to f$ uniformly on [a,b] Then prove that $\int_a^b f\, d\alpha = \lim_{n\to\infty} \int_a^b f_n d\alpha$

c) Show that the series $\sum_{n=1}^{\infty} \frac{nx}{1+n^2x^2}$ does not converge uniformly on [0, 1] **(6+6+4)**

4. a) State and prove stone weierstrass theorem.

b) State the following

i) Exponential function

ii) Logarithmic function **(12+4)**

5. a) Show that $\int_a^{\infty} \frac{1}{x^p dx}$ convergence if and only if p>1 where a>0.

b) If g(x) is continuous and decreasing on $[a,+\infty]$, $\lim_{x\to\infty} g(x) = 0$ and $\int_a^{\infty} g(x)dx$ diverges then P.T $\int_a^{\infty} g(x)|\sin x|dx$ diverges.

c) Show that $\int_0^{\infty} \frac{e^{-pt}-e^{-rt}}{t} dt = \log\left(\frac{q}{p}\right), O < p < q$ **(6+6+4)**

6. a) State and prove Euler - Poisson intigral and show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.

b) Let f be integrable and g be non-negative decreasing and continuous function on [a, b] Then show that $\int_a^b f\, g = g(c)\int_a^b f$, $\exists\, C \in [a,b]$ **(8+8)**

7. a) Let $f : R^2 \to R$ be defined by $f(x,y) = \begin{cases} xy\dfrac{x^2y^2}{x^2+y^2} & if\ (x,y) \neq (0,0) \\ 0 & ,\ if\ (x,y) = (0,0) \end{cases}$

then show that f is continuous at (0,0)

b) Find θ_1 and θ_2 from the function $f(x,y) = x^2 + y^2 + x^3$ at $(x_0, y_0) = (0.0)$

c) State young's theorem. **(7+7+2)**

8. a) Express $f(x,y) = x^2y + 3y - 2$ in powers of (x-1) and (y+2)

b) Define local maxima and local minima and show that $f(x,y) = 2x^4 - 3x^2y + y^2$ has neither a maximum nor a minimum at (0.0).

c) State contraction principle.

(7+7+2)
