MMDSC-2.2

Sl.No. : **Total No. of Pages : 3**

II Semester M.Sc. Degree Examination, May/June - 2023
(CBCS, SLM Scheme)
MATHEMATICS
Real Analysis-II

Time : 3 Hours **Max. Marks : 80**

Instructions: ***1) Answer any five full questions.***
2) All question carry equal marks.

Q1) a) A function *f* is Riemann-steiltjc's integrable on [a,b] if and only if for every $\in > 0$ there exists a partition P, such that $U(P, f, \alpha) - L(P, f, \alpha) < \in$

b) If $f \in \mathbb{R}(\alpha) \text{on} [a,b]$, then show that $-f \in \mathbb{R}(\alpha) \text{on} [a,b]$ and $\int_a^b (-f)\, d\alpha = -\int_a^b f\, d\alpha$.

c) If $f \in \mathbb{R}(\alpha)$ on $[a,b]$ and if $|f(x)| < \varepsilon$ for all $x \in [a,b]$, the $\left|\int_a^b f d\alpha\right| \leq M(\alpha(b) - \alpha(a))$.

[6+6+4]

Q2) a) If *f* is a bounded real function on $[a,b]$ and α is monotonically increasing on $[a,b]$ such that α' is Riemann integration on, $[a,b]$ then $f \in \mathbb{R}(\alpha)$ on $[a,b]$ if and only if $f\alpha'$ is Riemann integration on [a,b] and $\int_a^b f\, dx = \int_a^b f(x)\alpha'(x)\, dx$.

b) State and prove Fundamental theorem of calculus.

c) Show that a function of bounded variation is always bounded

[6+6+4]

Q3) a) State and prove cauchy criterion for Uniform convergence.

b) If α is monotonically increasing on [a,b]. Suppose $f_n \in \mathbb{R}(\alpha)$ on [a,b], for n=1,2,3,...., and suppose f_n converges uniformly to f on [a,b], then $f \in \mathbb{R}(\alpha)$ on $[a,b]$ and $\int_a^b f d\alpha = \lim_{n\to\infty} \int_a^b f_n d\alpha$

c) Prove that the sequence $\left\{\frac{x}{1+nx^2}\right\}. x \in \mathbb{R}$ is uniformly convergent on any closed interval $[a,b]$.

[6+6+4]

Q4) a) State and prove stone. Weierstrass theorem.

b) Show that the logarithm function $L(x)=nL(x)$, $x>0$, rational n.

[12+4]

Q5) a) Let f be a real bounded function defined on $[a,b]$ and α be monotonically increasing on [a,b], then $\int_{-a}^{b} f\, d\alpha \le \int_{a}^{-b} f\, d\alpha$

b) If P* is a refinement P, then prove that $L(P, f, \alpha) \le L(P^*, f, \alpha)$ and $U(P^*, f, \alpha) \le U(P, f, \alpha)$.

c) Show that a function of bounded variation is always bounded.

[6+6+4]

Q6) a) Let $f(x) > 0$ and $g(x) > 0$ for all $x \ge a$. If $\lim_{x\to a} \frac{f(x)}{g(x)} = l$ (finite) and $\int_a^b g(x)dx$ Converges, then $\int_a^b f(x)\,dx$ converges.

b) Show that with example if g is continuous and decreasing for $a \le x < \infty$ and $\lim_{x\to\infty} g(x) = 0$, then $\int_a^b g(x) \sin x\, dx$ converges.

c) Show that $\int_o^\infty \frac{e^{-pt} - e^{-qt}}{t} dt = \log\left(\frac{q}{p}\right), 0 < p < q$

[5+6+5]

Q7) a) Prove that $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$.

b) Let f and $\frac{\partial f}{\partial y}$ be continuous in $[a,b] \times [c,d]$. Then f is and $\frac{d}{dx}\{\phi(y)\} = \int_a^b \frac{\partial f(x,y)}{\partial y} dx$ holds.

c) If $f(x,y) = \begin{cases} xy \tan\left(\frac{y}{x}\right), & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0), \end{cases}$ then show that $xf_x + y f_y = 2f$.

[6+6+4]

Q8) a) State and prove young's theorem.

b) State and prove Taylor's theorem for a function of one variable.

c) Find the derivative of the function $y = \phi(x)$ at $x=1$ defined implicitly by the equation $e^{-x\cos y} + \sin y = e$

[5+8+3]