MMDSC-2.2

Sl.No. :

Total No. of Pages : 3

Second Semester M.Sc. Degree Examination, May - 2026
(SLM Scheme) (CBCS)
MATHEMATICS
Real Analysis - II

Time : 3 Hours **Max. Marks : 80**

Instruction : Answer any Five questions.

Q1) a) If P^* is a refinement of P, then prove that $L(p, f, \alpha) \leq L(p^*, f, \alpha)$ and $U(p^*, f, \alpha) \leq U(p, f, \alpha)$.

b) Show that $f(x) = \begin{cases} 1 & \text{if } x \text{ is a rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$

is not Riemann - Stieltje's integrable on $[a, b]$ for any monotonically increasing function.

c) If $f \in \mathbb{R}(\alpha)$ on $[a, b]$ and if $|f(x)| < \varepsilon$ for all $x \in [a, b]$ then prove that $\left| \int_a^b f d\alpha \right| \leq M(\alpha(b) - \alpha(a))$.

[6 + 6 + 4]

Q2) a) If $f \in \mathbb{R}(\alpha_1)$ on $[a, b]$ and $f \in \mathbb{R}(\alpha_2)$ on $[a, b]$ then show that $f \in \mathbb{R}(\alpha_1 + \alpha_2)$ on $[a, b]$ and further show that $\int_a^b f\, d(\alpha_1 + \alpha_2) = \int_a^b f\, d\alpha_1 + \int_a^b f\, d\alpha_2$.

b) Let $f \in \mathbb{R}(\alpha)$ on $[a, b]$ and let $g(x) = \int_a^x f(t)dt$ for all $x \in [a, t]$. Then prove that $g(x)$ is continuous on $[a, b]$ further if f is continuous at $x_o \in [a, b]$ then g is differentiable at x_o and further show that $g'(x_o) = f(x_o)$.

[8 + 8]

Q3) a) Prove that product of two functions of bounded variation functions is also of bounded variation.

b) Let α be monotonically increasing on $[a, b]$ suppose $f_n \in R(\alpha)$ on $[a, b]$ for $n = 1, 2, 3, ...$ and suppose $f_n \to f$ uniformly on $[a, b]$ then prove that $\int_a^b f \, d\alpha = \lim_{n\to\infty} \int_a^b f_n d\alpha$.

c) Evaluate $\int_0^3 x dx([x] - x)$.

[7 + 6 + 3]

Q4) a) State and prove Stone Weierstrass theorem.

b) Show that the exponential function $E(q) = e^q$, for every rational number q.

[12 + 4]

Q5) a) Find the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$.

b) If $f' [a, b] \to \mathbb{R}$, $f(x) = K$ and $\alpha : [a, b] \to \mathbb{R}$ be monotonically increasing function. Then prove that $f \, \mathbb{R}(\alpha)$ on $[a, b]$.

c) Show that $\int_0^{\infty} \frac{e^{-pt} - e^{-qt}}{t} dt = \log\left(\frac{q}{p}\right), 0 < p < q$.

[6 + 6 + 4]

Q6) a) Prove that $\beta(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x + y)}, x > 0, y > 0$.

b) Show that Euler - Poisson integral $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$.

c) Evaluate $\int_0^1 \log(\Gamma(x)) dx$.

[6 + 4 + 6]

Q7) a) Let f and $\frac{\partial f}{\partial y}$ be continuous in $[a, b] \times [c, d]$ then show that f is differentiable and $\frac{d}{dx}\phi(y) = \int_a^b \frac{\partial f(x,y)}{\partial y}.dx$ holds.

b) Let $f : R^2 \to R$ be defined by $f(x,y) = \begin{cases} xy.\frac{x^2y^2}{x^2+y^2} & if \quad (x,y) \neq (0,0) \\ 0 & if \quad (x,y) = (0,0) \end{cases}$

Then show that f is continuous at (0, 0).

[8 + 8]

Q8) a) State and prove Euler's theorem for homogeneous function.

b) Let f be a real valued continuously differentiable function defined on an open subset G of R^2. Suppose $(x_0, y_0) \in G$, such that

i) $f(x_0, y_0) = 0$

ii) $D_2 f(x_0, y_0) \neq 0$

and ϕ is differentiable on $(x_0 - h, x_0 + h)$ then prove that $\phi'(x) = \frac{-D_1 f(x,\phi(x))}{D_2 f(x,\phi(x))}$.

[8 + 8]

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