MMDSC-2.2

Sl. No.

II Semester M.Sc. Degree Examination, May/June - 2025
MATHEMATICS
Real Analysis - II
(CBCS SLM Scheme)

Time : 3 Hours **Maximum Marks : 80**

Answer any **Five** full questions.

1. a) Let f be a real bounded function defined on $[a, b]$ and let α be monotonically increasing on $[a, b]$ then prove that $\int_{-a}^{b} f\, d\alpha \le \int_{a}^{-b} f\, d\alpha$.

b) If $\lim_{\|P\|\to 0} S(P,Q,f,\alpha)$ exists then $f \in R(\alpha)$ on [a, b] and prove that

$$\lim_{\|P\|\to 0} S(P,Q,f,\alpha) = \int_{a}^{b} f\, d\alpha .$$

c) Show that $f(x) = \begin{cases} 1 & if\ x\ is\ a\ rational \\ 0 & if\ x\ is\ irrational \end{cases}$

is not Riemann-Stieltjes integrable on $[a, b]$ for any monotonically increasing function $\alpha(x)$. **(5+5+6)**

2. a) If $f \in R(\alpha)$ on $[a, b]$ and if $a < c < b$ then $f \in R(\alpha)$ on $[a, c]$ and $f \in R(\alpha)$ on $[c, b]$ further show that $\int_{a}^{b} f\, d\alpha = \int_{a}^{c} f\, d\alpha + \int_{c}^{b} f\, d\alpha$.

b) Let $f \in R(\alpha)$ on $[a, b]$ and let $g(x) = \int_{a}^{x} f(t)\, dt\ \forall\, x \in [a,t]$ then prove that $g(x)$ is continuous on $[a, b]$. Further if f is continuous at $x_0 \in [a,b]$, then g is differentiable at x_0 and $g'(x_0) = f(x_0)$. **(6+10)**

[P.T.O.

3. a) State and prove Fundamental theorem of calculus.

b) Prove that product of two functions of bounded variation functions is also of bounded variation. **(8+8)**

4. a) If r' is continuous function on $[a, b]$ then r is rectifiable then prove that $a(r) = \int_a^b \|r'(t)\| dt$.

b) State and prove Cauchy Criterion for uniform convergence. **(8+8)**

5. a) Let α be monotonically increasing on [a, b]. Suppose $f_n \in R(\alpha)$ on $[a, b]$ for $n = 1$, 2, 3, ... and suppose $f_n \to f$ uniformly on $[a, b]$ then prove that $f \in R(\alpha)$ on $[a, b]$ and $\int_a^b f\, d\alpha = \lim_{n\to\infty} \int_a^b f_n\, d\alpha$.

b) State and prove Stone Weierstrans theorem. **(6+10)**

6. a) Find the radius of convergence of the series $\sum_{n=o}^{\infty} \frac{x^n}{n!}$.

b) If f and g are continuous on (a, b) and are unbounded at a 'a' and if $0 \le f(x) \le g(x)$ (a, b) and $\int_{a^+}^b g(x)\, dx$ d converges then prove that $\int_{a^+}^b f(x)\, dx$ converges. **(8+8)**

7. a) If g is continuous and decreasing for $a \le x < \infty$ and $\lim_{x\to\infty} g(x) = 0$ then prove that $\int_a^{\infty} g(x) \sin x.dx$ converges.

b) Prove that $\beta(x, y) = \frac{\overline{|(x)}\ \overline{|(y)}}{\overline{|(x+y)}}$, $x > 0, y > 0$.

c) Let f be integrable and g be non-negative, decreasing and continuous on $[a, b]$ then prove that $\int_a^b fg = g(c) \int_a^b f \quad \forall c \in [a, b]$. **(4+6+6)**

8. a) Let $f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}} & , \ (x,y) \neq (0,0) \\ 0 & , \ (x,y) = (0,0) \end{cases}$ then show that $\lim\limits_{(x,y)\to(0,0)} f(x,y) = 0$.

b) Express $f(x,y) = x^2y + 3y - 2$ in power of (x-1) and (y+2).

c) Let f be a Real valued continuously differentiable function defined on an open subset G of R^2. Suppose $(x_0, y_0) \in G$, such that

i) $f(x_0\, y_0) = 0$ ii) $D_2 f(x_0, y_0) \neq 0$

and ϕ is differentiable on $(x_0 - h,\ x_0 + h)$ then prove that $\phi'(x) = \dfrac{-D_1 f(x, \phi(x))}{D_2 f(x, \phi(x))}$.

(4+4+8)
