MMDSC-2.2

Sl.No. : **Total No. of Pages : 3**

Second Semester M.Sc. Degree Examination, November - 2024

(SLM Scheme) (CBCS)

MATHEMATICS

Real Analysis - II

Time : 3 Hours **Max. Marks : 80**

Instruction : Answer any five full questions.

Q1) a) A function f is Riemann - Stieltje's integrable on $[a, b]$ if and only of for every $\in > 0$ there exists a partition P such that $U(p, f, \alpha) - L(p, f, \alpha) < E$.

b) If p^* is a refinement p, then prove that $L(p, f, \alpha) \leq L(p^*, f, \alpha)$ and $U(p^*, f, \alpha) \leq U(p, f, \alpha)$.

[8+8]

Q2) a) If $f \in R(\alpha_1)$ on $[a, b]$ and $f \in R(\alpha_2)$ on $[a, b]$ then P.T $f \in R(\alpha_1 + \alpha_2)$ on $[a, b]$ and $\int_a^b f\, d(\alpha_1 + \alpha_2) = \int_a^b f\, d\alpha_1 + \int_a^b f\, d\alpha_2$.

b) If $f \in R(\alpha)$ on $[a, b]$ then show that $-f \in R(\alpha)$ on $[a, b]$ and $\int_a^b (-f)\, d\alpha = -\int_a^b f\, d\alpha$.

[8+8]

Q3) a) State and prove first mean value theorem for Riemann - stieltjes integrals.

b) Let $f \in R(\alpha)$ on $[a, b]$ and let $g(x) = \int_a^x f(t)dt \; \forall \; x \in [a,t]$ then $g(x)$ is continuous on $[a, b]$. Further if f is continuous at $x_0 \in [a, b]$ then g is differentiable at x_0 and prove that $g'(x_0) = f(x_0)$.

[8+8]

Q4) a) State and prove Jordan theorem.

b) Let f and α be complex valued functions of bounded variation on $[a, b]$ and let be also continuous an $[a, b]$ then P.T.

$$\int_a^b f\, d\alpha = f(b)\alpha(b) - f(a)\alpha(a) - \int_a^b \alpha df\ .$$

c) Evaluate $\int_0^3 x\, d([x]-x)$.

[4+8+4]

Q5) a) State and prove Cauchy criterion for uniform Convergence.

b) Show that the series $\sum_{n=1}^{\infty} \frac{nx}{1+n^2x^2}$ does not converge uniformly on [0.1].

c) Let α be monotonically increasing on [a, b] suppose $f_n \in R(\alpha)$ on $[a, b]$ for $n = 1, 2, 3, ...$ and suppose $f_n \to f$ uniformly on $[a, b]$ then $f \in R(\alpha)$ on $[a, b]$ and $\int_a^b f d\alpha = \lim_{n\to\infty} \int_a^b f_n d\alpha$.

[6+4+6]

Q6) a) State and prove Stone - Weierstrass approximation theorem.

b) If f is continuous on $[a, +\infty)$ and $\int_a^{\infty} f(x)dx$ converges absolutely then prove that $\int_a^{\infty} f(x)$ is convergent.

c) Find the radius of convergence of the series $\sum_{n=0}^{\infty} x^n$.

[8+4+4]

Q7) a) If f is continuous on $a < x \le b$ and $(x - a)^p f(x) = A$, where $0 < p < 1$, then prove that $\int_{a^+}^b |f(x)|dx$ converges.

b) Show that Euler - Poisson integral $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$.

c) Suppose that $\int_a^{\infty} f(x,y)\,dx$ converges uniformly for $y \in [c, d]$ and $g(x, y)$ is decreasing for each fixed y and is bounded then P.T. $\int_a^{\infty} f(x,y)\,g(x,y)\,dx$ converges uniformly on $[c, d]$.

[4+4+8]

Q8) a) Show that $\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$ does not exist.

b) State and prove Euler's theorem for homogeneous function.

c) Find the derivative of the function $y = \phi(x)$ at $x = 1$ defined implicitly by the equation $e^{-x\cos y} + \sin y = e$.

[4+8+4]

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