MMDSC-2.3

Sl. No.

II Semester M.Sc. Degree Examination, December - 2023
MATHEMATICS
Complex Analysis - II
(CBCS Scheme)

Time : 3 Hours **Maximum Marks : 80**

***Instructions to Candidates*:**

1) ***Answer any Five full questions.***
2) ***All questions carry equal marks.***

1. a) State and prove Rouche's theorem and hence prove the fundamental theorem of algebra.

b) Determine the nature of singularity of $f(z) = \frac{\sin \pi z}{(z-1)^3}$ at $z_0 = 1$, compute Res(f(x),1).

c) Compute the residue at each singularity of $\frac{Z^4 + Z^2}{\text{Sin}\,\pi z}$ **(8+4+4)**

2. a) If $\lim_{z \to a} (z\text{-}a) f(z) = A$ and if c, is the arc $\theta_1 \le \theta \le \theta_2$ if the circle |Z-a|=r, then prove that $\lim_{x \to 0} \int_c f(z)dz = iA(\theta_2 - \theta_1)$.

b) Evalute the following integrals **(8+8)**

i) $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+4)}$.

ii) $\int_0^{\infty} \frac{\sin cx}{x} dx,\ c > 0$.

[P.T.O.

3. a) Prove that the infinite product $\prod(1+a_n)$ is convergent if the two series $\sum_{a_n}$ and $\sum|a_n|^2$ are both convergent. **(8+8)**

b) Prove legendere's duplication formula for gamma function.

4. a) State and prove the weierstrass factorization theorem.

b) State and prove the Hadamard's theorem. **(8+8)**

5. a) Prove that any bilinear transformation other than identity has at most two fixed points.

b) Prove that any bilinear transformation can be expressed as a product of translation, rotation, magnification or contraction and inversition. **(8+8)**

6. a) Show that if the bilinear transformation has distinct invariant points p and q the transformation may be in the form. $\frac{w-P}{w-q}=K\frac{(Z-p)}{(Z-q)}$.

b) State and prove reflection principal. **(8+8)**

7. a) Prove that a non constant elliptic function has equally many poles as it has zero.

b) State and prove the Hurwitz's theorem.

c) State and prove maximum modulus theorem. **(4+6+6)**

8. a) Show that the transformation given by P = 2 cot p. $Q=\log\left(\frac{\sin p}{2}\right)$ is a canonical transformation.

b) Prove that $\sin\pi z\prod_{n=1}^{\infty}\left(1-\frac{z^2}{n^2}\right)$. **(8+8)**
