MMDSC - 2.3

Sl.No. : **Total No. of Pages : 2**

II Semester M.Sc. Degree Examination, May/June - 2023
(CBCS, SLM Scheme)
MATHEMATICS
Complex Analysis - II

Time : 3 Hours **Max. Marks : 80**

Instructions: ***1)*** ***Answer any five full questions.***
2) ***All questions carry equal marks.***

Q1) a) Prove that a harmonic function in a domain D possesses a harmonic (onjugate in D if and only if D is simply connected.

b) State and prove poisson integral formula.

[8+8]

Q2) Evaluate the following integrals.

a) $\int_{|z|=2\pi} \tan z \, dz.$

b) $\int_{|z|=1} \frac{(2z+1)^2}{\tan^{-1}\left(\frac{iz}{2}\right)} dz.$

c) $\int_0^{\pi} \sin^{2n}\theta \, d\theta,$ for n is a positive integer.

[4+4+8]

Q3) a) Prove that the infinite product $\prod_{n=1}^{\infty}(1+a_n)$ is convergent if and only if the series $\sum_{n=1}^{\infty}\log(1+a_n)$ is convergent.

b) State and prove Hadamard's theorem.

[8+8]

Q4) a) State and prove Legendre duplication formula.

b) State and prove Jensen's formula for an entire function.

[8+8]

Q5) a) Find the bilinear transformation which maps the points –1, 0, 1 onto points 0, i, $3i$.

b) Find the fixed points and the normal form of the bilinear transformation $w = \dfrac{z}{Q - z}$.

c) Prove that the sufficient condition for $w = f(z)$ to represent a conformal mapping.

[5+5+6]

Q6) a) Show that the resultart (or product) of two bilinear transformation is a bilinear transformation.

b) S.T. the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points z_1, z_2, z_3, z_4 lie on a circle or on a straight line.

c) S.T. the cross ratios remains invariant under a bilinear transformation.

[5+5+6]

Q7) a) State and prove Riemann mapping theorem.

b) Find the complex form of Fourier series of $f(x) = e^{-x}$ in $-1 \le x \le 1$.

[10+6]

Q8) a) Solve the equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square with sides $x = 0 = y$, $x = 3 = y$ with $u = 0$ on the boundary and mesh length = 1.

b) Define simply periodic function.

c) Show that threre exist a basis (w_1, w_2) such that the ratio $\tau = w_1, w_2$ satisfies the following conditions.

i) Im $\tau > 0$

ii) $-\dfrac{1}{2} < \text{Re}\,\tau \le \dfrac{1}{2}$

iii) $|\tau| \ge 1$

iv) Re $\tau \ge 0$ if $|\tau| = 1$.

[7+2+7]

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