**MMDSC-2.3**

Sl.No. :

**Total No. of Pages : 2**

# Second Semester M.Sc. Degree Examination, May - 2026
## (SLM Scheme) (CBCS)
## MATHEMATICS
## Complex Analysis - II

**Time : 3 Hours** **Max. Marks : 80**

**Instructions : 1) Answer any Five questions.**
**2) All questions carry equal marks.**

***Q1)*** a) State and prove Rouche's theorem.

b) Evaluate $\int_{-\infty}^{\infty} \frac{1}{(x^2+1)(x^2+4)}\,dx$.

c) Find the residue of $f(z) = \frac{1-\cos z}{z^3}$ at $z = 0$.

**[7 + 7 + 2]**

***Q2)*** a) State and prove Argument Principle.

b) If $f(z) \to 0$ uniformly as $z \to \infty$ and $f(z)$ is meromorphic in the upper half plane then prove that $\lim_{R\to\infty} \int_{\gamma} e^{icz} f(z)dz = 0$, ($c > 0$) where $\gamma$ denotes the semicircle $|z| = R$ and $im(z) > 0$.

**[8 + 8]**

***Q3)*** a) Prove that a real valued continuous function in a plane open set $D$ is harmonic in $D$ if and only if it has the mean value property in $D$.

b) If $u_1(x, y)$ and $u_2(x, y)$ are any two harmonic functions in a region $\Omega$ and $\gamma$ be any cycle homologous to zero in a region $\Omega$ then, show that $\int_{\gamma} u_1^* du_2 - u_2^* du_1 = 0$.

c) Define conjugate differential of $du$ where $u$ is harmonic function.

**[7 + 7 + 2]**

***Q4)*** a) State and prove Schwarz's theorem.

b) State and prove reflection principle for harmonic function.

c) State Harnack's inequality.

**[7 + 7 + 2]**

***Q5)*** a) Define uniform convergence of a sequence of function. Suppose $\{f_n\}$ converges uniformly to a function $f$ on a set $S$. If each $f_n$ is continuous on $S$ then prove that $f$ is continuous on $S$. Furthermore prove that $\int_\gamma f(z)dz = \lim_{n\to\infty} \int_\gamma f_n(z)dz$ for every piecewise smooth path $\gamma$ in $S$.

b) State and prove Weierstrass M-test for series of analytic function.

c) Define pointwise convergence and uniform convergence of a sequence of function.

**[7 + 7 + 2]**

***Q6)*** a) State and prove Laurent's theorem.

b) Find the Taylor series development of $f(z) = \dfrac{z}{e^z - 1}$ upto five terms.

c) Find the Taylor series expansion of $f(z) = e^z$ centered at origin.

**[10 + 4 + 2]**

***Q7)*** a) State and prove Jensen's formula.

b) Prove that the infinite product $\prod_{n=1}^{\infty}(1+a_n)$ is convergent if the two series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} |a_n|^2$ are both convergent.

c) If $\prod_{n=1}^{\infty} a_n$ converges then show that $\lim_{n\to\infty} a_n = 1$.

**[7 + 7 + 2]**

***Q8)*** a) State and prove Hadamard's three circle theorem.

b) Show that $\Gamma(1-z)\Gamma(z) = \dfrac{\pi}{\sin \pi z}$.

c) Show that $\Gamma(z+1) = z\Gamma(z)$.

**[10 + 4 + 2]**

⌘⌘⌘