MMDSC-2.3

Sl.No. :

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II Semester M.Sc. Degree Examination, May - 2024

(CBCS, SLM Scheme)

MATHEMATICS

Complex Analysis - II

Time : 3 Hours **Max. Marks : 80**

Instructions: ***1)*** **Answer any five full questions.**

2) All questions carry equal marks.

Q1) a) State and prove Cauchy's Residue theorem.

b) State and prove the Argument Principle.

c) Find residue of $f(z)=\dfrac{z^2+16}{(z-i)^2(z+3)}$ at its singularities.

[7+7+2]

Q2) a) Evaluate $\int_0^{2\pi}\dfrac{1}{2+\cos\theta}\,d\theta$.

b) Evaluate $\int_0^{\infty}\dfrac{\sin mx}{x\left(x^2+a^2\right)^2}\,dx, m>0, a>0$.

c) Show that $p(z)=z^5+z^3+5z^2+2$ has three zeros (counting multiplicities) in the annular region $1<|z|<2$.

[7+7+2]

Q3) a) Prove that the infinite product $\prod_{n=1}^{\infty}(1+a_n)$ is convergent if and only if the series $\sum_{n=1}^{\infty}\log(1+a_n)$ is convergent, where the principal value of the logarithm being taken in each case.

b) State and prove general principle of convergence for infinite products.

c) Express gamma function as an infinite product.

[7+7+2]

Q4) a) Prove that the function $\xi(z)$ satisfies the function equation $\xi(z)=2^{z}\pi^{z-1}\sin\left(\frac{\pi z}{2}\right)\Gamma(1-z)\xi(1-z)$.

b) Prove that, $\xi(1-z)=2^{1-z}\pi^{-z}\cos\left(\frac{\pi z}{2}\right)\Gamma(z)\xi(z)$.

[8+8]

Q5) a) State and prove the conformal mapping theorem.

b) Let D be domain in the complex plane and let $f:\mathrm{D}\to\mathrm{C}$ be a differentiable. Show that the following three statements concerning a point z_0 of D are equivalent.

i) f is differentiable at z_0.

ii) f is isogonal at z_0 and $\mathrm{J}_f(z_0)>0$.

iii) f is conformal at z_0.

[8+8]

Q6) a) Prove that a bilinear transformation maps the family of circles and lines into the family of circles and lines.

b) Prove that any bilinear transformation other than identify has atmost two fixed points.

c) Find the bilinear transformation which maps the points 0, *-i,* -1 onto *i,* 1, 0 respectively.

[7+7+2]

Q7) a) Prove that the number of zeros or poles of an elliptic function in any cell is finite.

b) Prove that the sum of the residues of an elliptic function is zero.

c) Prove that a discrete module consists either of zero alone of the intergral multiples *nw* of a single complex number $w \neq 0$ or of all linear combinations $n_1w_1+n_2w_2$ with integral coefficients of two numbers $w_1 w_2$ with non-real ratio w_1 / w_2.

[5+5+6]

Q8) a) Write a note on unimodular transformations.

b) Prove that $\sin \pi z = \pi z \prod_{n=1}^{\infty} \left(1 - \frac{z^2}{n^2}\right)$.

[10+6]