MMDSC-2.3

Sl. No.

II Semester M.Sc. Degree Examination, May/June - 2025
MATHEMATICS
Complex Analysis - II
(CBCS SLM Scheme)
Course - II

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

Answer any Five full questions.

1. a) Find the residue of $\dfrac{\cot \pi z}{(z-a)^2}$.

b) State and prove argument principle theorem.

c) If $\lim_{z\to a}(z-a)f(z) = \text{A}$ and if C is the arc $\theta_1 \le \theta \le \theta_2$ of the circle $|z-a| = r$, then prove that $\lim_{r\to 0}\int_c f(z)dz = iA(\theta_2 - \theta_2)$. **(5+6+5)**

2. a) State and prove Schwartz's theorem.

b) Prove that $\int_0^{2\pi} \dfrac{\theta}{a+b\cos\theta} = \dfrac{2\pi}{\sqrt{a^2+b^2}}$, $a > b > 0$. **(8+8)**

3. a) Show that the infinite product $\prod(1+f_n(z))$ would be uniformly convergent in S if the series $\sum|f_n(z)|$ is uniform convergent in S.

b) Test the convergence of sequence $3+(-1)^n$.

c) Check weather its uniform convergent sequence $\sum_{n=1}^{\infty}\dfrac{x}{(1+nx^2)n}$. **(6+5+5)**

4. a) State and prove Jensen's formula.

b) Represent $\sin \pi z$ in form of canonical product.

c) Evaluate $\int_0^{\infty} e^{-5x}x^4 dx$. **(6+5+5)**

[P.T.O.

5. a) Prove that the transformation $w = ze^{i\frac{\pi}{4}}$, the line $y = 0$ is transformed into the line.

b) Find all the bilinear transformation which transforms the unit circle $|z| \le 1$ into the unit circle $|w| \le 1$. **(8+8)**

6. a) State and prove open mapping theorem.

b) Find the mobius transformation which maps 0, 1, ∞ into 1, i, -1 respectively. **(8+8)**

7. a) Prove that a non-constant elliptic function has equally many poles as it has zeros.

b) Show that the transformation given by $P = q \cot p, Q = \log\left(\frac{\sin p}{q}\right)$ is a canonical transformation. **(8+8)**

8. a) Prove that $\sinh z = z\prod_{n=1}^{\infty}\left(1 - \frac{z^2}{n^2\pi^2}\right)$.

b) Find the complex form of fourier series of $f(x) = e^{-x}$ in $-1 \le x \le 1$. **(8+8)**
