MMDSC-2.3

Sl.No. :

Total No. of Pages : 2

Second Semester M.Sc. Degree Examination, November - 2024

(CBCS) (SLM Scheme)

MATHEMATICS

Complex Analysis - II

Time : 3 Hours **Max. Marks : 80**

Instruction : Answer any five full questions.

Q1) a) Find the residue of $\frac{z^3}{(z-1)^4(z-2)(z-3)}$ *at* $z=1$.

b) State and prove Residue theorem.

c) If C is an arc $\theta_1 \le \theta \le \theta_2$ of the circle $|z| = R$ and if $\lim_{n\to\infty} z f(z) = A$, then prove that $\lim_{R\to\infty} \int_C f(z)dz = iA(\theta_2 - \theta_2)$.

[5 + 6 + 5]

Q2) a) Prove that $\int_0^{2\pi} \frac{\sin^2\theta}{a+b\cos\theta} d\theta = \frac{2\pi}{b^2}\left(a - \sqrt{a^2-b^2}\right)$, if $a > b > 0$.

b) State and prove Poisson's formula.

[8 + 8]

Q3) a) Prove that infinite products $\prod(1+b_n)$ and $\prod(b_n)$ convergence if and only if the positive series $\sum b_n$ converges.

b) Test the convergence of sequence $\frac{n^2+n}{2n^2-n}$.

c) If f_n is real and non-negative then the series $\sum f_n$ and the $\prod(1+a_n)$ converge or diverge together.

[6 + 5 + 5]

Q4) a) State and prove Hadmard's theorem.

b) Show that $\int_0^\infty x^m e^{-ax^n}\,dx = \dfrac{\Gamma\left(\dfrac{m+1}{n}\right)}{na^{\left(\frac{m+1}{n}\right)}}$.

[10 + 6]

Q5) a) Show that $w = \dfrac{i - iz}{1 + z}$ transforms the circle $|z| = 1$ onto the real axis of the w-plane.

b) Find all the mobius transformations which transform the half plane $I(z) \geq 0$ into circle $|w| \leq q$.

[8 + 8]

Q6) a) If $U \subset C$ is open and connected and that f : U $\rightarrow$ C is non constant and holomarphic then prove that f(U) is open.

b) Find the bilinear transformation which maps the point $z = 0, -1, \infty$ into the points $w = -1, -2, -i$ respectively.

[8 + 8]

Q7) a) Prove that the sum of the residues of an elliptic function is zero.

b) Show that the transformation given by $P = (2q)^{\frac{1}{2}} e^{-\alpha} \sin p, Q = (2q)^{\frac{1}{2}} e^{\alpha} \cos p$ is a canonical transformation.

[8 + 8]

Q8) a) Prove that $\sin \pi z = \pi z \prod_{n=1}^{\infty} \left(1 - \dfrac{z^2}{n^2}\right)$.

b) Show that $f(z) = z^2$ is holomarphic on C.

c) Express $z = 1 - \sqrt{3i}$ in the form $z = re^{i\theta}$.

[6 + 5 + 5]

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