MMDSC-2.4

Sl. No.

II Semester M.Sc. Degree Examination, December - 2023

MATHEMATICS

Partial Differential Equations

(CBCS Scheme)

Time : 3 Hours **Maximum Marks : 80**

***Instructions to Candidates*:**

1) Answer any **Five** full questions.
2) All questions carry **Equal** marks.

1. a) Eliminate arbitrary constants a and b from $z=(x-a)^2+(y-b)^2$ to form the partial differential equation. **(5+5+6)**

b) Solve $p\cot x + q\cot y = \cot z$.

c) Find the orthogonal trajectories of the family of curves $y^2=4cx$ where c is an arbitrary constant.

2. a) Solve the Pfaffian equation $xdy-ydx=0$. **(5+5+6)**

b) Solve the cauchy problem by the method of characteristics $u_x+u_y+u=1$ with the initial data $u=\sin x$ on $y=x+x^2$.

c) Solve $(D^3-3D^2D_1+4D_1^3)z=e^{x+2y}$.

3. a) Find the complete integral of the equation $pq-p-q=0$. **(6+6+4)**

b) Solve $(p^2+q^2)y=qz$ by charpit's method.

c) Classify the partial differential equation $z_{xx}+z_{yy}=0$.

4. a) Classify the equation $\frac{\partial^2 u}{\partial x^2} = x^2 \frac{\partial^2 u}{\partial y^2}$ and reduce it to the canonical form. **(10+6)**

b) Construct an adjoint to the laplace operator $L(u)=u_{xx}+u_{yy}$.

[P.T.O.

5. a) State and prove Poisson integral formula. **(10+6)**

b) Solve the Boundary value problem $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ if $u(o,y) = 8e^{-3y}$.

6. a) If $f(t)$ is any continuous function then show that $\int_{-\infty}^{\infty} \delta(t-a) f(t)dt = f(a)$. **(8+8)**

b) Solve by Green's function method

$$y'' + 4y = f(x),\ \forall x \in \left[0, \frac{\pi}{2}\right], y(0) = y'\left(\frac{\pi}{2}\right) = 0.$$

7. a) Derive D'Alembert solution of one dimensional wave equation. **(10+6)**

b) Find Eigen values and Eigen functions of

$$y'' + \lambda y = 0,\ \ 0 \le x \le \pi,\ \ y(0) = 0,\ \ y(\pi) = 0$$

8. a) Use Duhamel's principle to solve heat equation problem described by **(10+6)**

$$u_t(x,t) = ku_{xx}(x,t) + f(x,t),\ -\infty < x < \infty,\ t > 0 \text{ and } u(x,0) = 0,\ \infty < x < \infty.$$

b) Solve by the method of separation of variables $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$.
