MMDSC - 2.4

Sl.No. : **Total No. of Pages : 2**

II Semester M.Sc. Degree Examination, June - 2023
(CBCS, SLM Scheme)
MATHEMATICS
Partial Differential Equations

Time : 3 Hours **Max. Marks : 80**

Instructions: ***1) Answer any five full questions.***
2) All questions carry equal marks.

Q1) a) Find a partial differential equation by eliminating a and b from $z = ax + by + a^2 + b^2$.

b) Solve $p\sqrt{x} + q\sqrt{y} = \sqrt{z}$.

c) Solve $u_{tt} = c^2 u_{xx}$, $-\infty < x < \infty$, $0 < t < \infty$.

[5+6+5]

Q2) a) Solve the cauchy problem $uu_x + uu_y$, $u(x, 0) = 0$, $0 \le x \le 1$.

b) Solve $\left(4D^2 + 12DD_1 + 9D_1^2\right)z = e^{3x+2y}$.

c) Find the complete integral of the equation $upq - p - q = 0$.

[5+5+6]

Q3) a) Solve $(p^2 + q^2)y = qz$ by carpit's method.

b) Solve $\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial r^2} = 0$.

[8+8]

Q4) a) Classify the partial differential equation $u_{xx} - u_{xy} + xy\, u_{yy} = 0$.

b) Classify the equation $\frac{\partial^2 u}{\partial x^2} = x^2 \frac{\partial^2 u}{\partial y^2}$ and reduce it to cannonical form.

[6+10]

Q5) a) Solve $2zx - px^2 - 2pxy + pq = 0$.

b) It the Neumann problem for a bounded region has a solution then prove that it is either unique or it differers from one another by a constant.

[8+8]

Q6) a) If the Dirichlet problem for a bounded region has a solution then prove that it is unique.

b) If f(t) is any continuous function then show that $\int_{-\infty}^{\infty} \delta(t-a) f(t) dt = f(a)$

[8+8]

Q7) a) Solve $y'' + \lambda y = 0, 0 < x < \mathrm{L}$ with $y(-\mathrm{L}) = y(\mathrm{L})$ and $y'(-\mathrm{L}) = y'(\mathrm{L})$ by using periodic Eigen value problem.

b) Derive D' Alembert solution of one dimensional wave equation.

[6+10]

Q8) a) Find the steady - state temperature of the rod. If the fixed temperature at the ends $x = 0$ and $x = \pi$ are w_1 and w_2 respectively.

b) State and prove poisson integral formula.

[8+8]

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