MMDSC-2.4

Sl.No. :

Total No. of Pages : 2

II Semester M.Sc. Degree Examination, May - 2024

(CBCS, SLM Scheme)

MATHEMATICS

Partial Differential Equations

Time : 3 Hours **Max. Marks : 80**

Instructions: ***1)*** **Answer any five full questions.**

2) All questions carry equal marks.

Q1) a) Find the partial differential equation by eliminating arbitrary constants $z = (x + a)(y + b)$.

b) Solve $p \tan x + q \tan y = \tan z$.

c) Find the orthogonal trajectories of the family of curves $x^2 + y^2 + c^2$. Where c is an arbitrary constant.

[5+5+6]

Q2) a) Find the Cauchy problem for $u_x + u_y + u = 1$ with initial data $u = \sin x$ on $y = x + x^2$.

b) Solve $\left(4D^2 + 12DD' + 9D'^2\right)Z = e^{3x+2y}$.

c) Solve Pfaffian equation $xdy - ydx = 0$

[5+5+6]

Q3) a) Find the complete integral of $px + qy = pq$.

b) Solve $p^2 + qy - u = 0$ by Charpit's method.

c) Solve $p^2x + q^2y = z$ by Jacobi's method.

[5+6+5]

Q4) a) Classify the partial differential equation $u_{xx} - u_{xy} + xy\, u_{yy} = 0$.

b) Classify the equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + 2\frac{\partial^2 u}{\partial y \partial x} = 0$ and reduce it to canonical form.

[8+8]

Q5) a) Prove that

$$u(r,\theta) = \frac{1}{2\pi}\int_0^{2\pi} \frac{\left(a^2 - r^2\right) f(\theta)}{\left[a^2 - 2ar\cos(\phi - \theta) + r^2\right]}\, d\phi.$$

b) For any continuous function $f(t)$, then prove that $\int_{-\infty}^{\infty} f(t)\delta(t)dt = f(0)$.

[12+4]

Q6) a) Show that if two dimensional Laplace equation $\nabla^2 u = 0$ is transformed by introducing plane polar co-ordinates r, θ defined by the relations $x = r\cos\theta$, $y = r\sin\theta$ it takes the form $u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$.

b) Solve the Boundary value problem $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ if $u(0, y) = 8e^{-3y}$.

[8+8]

Q7) a) Derive D' Alembert solution of one dimensional wave equation.

b) Solve $y'' + \lambda y = 0$, $0 < x < L$ with $y(-L) = y(L)$ and $y'(-L) = y'(L)$ by using periodic Eigen value problem.

[10+6]

Q8) a) Solve $\frac{d^2 y}{dx^2} = f(x) = 1, \quad \forall x \in [0,1]$ by Strum's Liouville problem using Green function method with boundary conditions $y(0) + y'(0) = 0$, $y(1) + y'(1) = 0$.

b) Prove that the Dirichlet problem for a bounded region has a unique solution.

[8+8]