MMDSC-2.5

Sl. No.

II Semester M.Sc. Degree Examination, December - 2023
MATHEMATICS
Discrete Mathematics
(CBCS Scheme)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Answer any **Five** full questions.
2. All questions carry equal marks.

1. a) Define logical connectives. Construct the truth tables for the following statements.
   i. Distributive laws **(8+4+4)**
   ii. De Morgan's laws

b) Explain the following:
   i. Rule of universal specification
   ii. Rule of existential specification

c) Test whether the following argument are valid.

$(p \to q) \wedge (r \to s) \wedge (p \vee r) / \therefore (q \vee s)$

2. a) Prove that $\sqrt{2}$ is irrational using the method of contradiction. **(4+6+6)**

b) Obtain the Disjunctive Normal Forms (DNF) and Conjunctive NORMAL Forms (CNF) of $p \wedge (p \to q)$

c) Obtain Principle Disjunctive Normal Forms (PDNF) of $(\neg p \vee \neg q) \to (\neg q \wedge r)$

3. a) Prove by mathematical induction that for all positive integers $n \geq 1$

$$1^2+2^2+3^2+.....+ n^2 = \frac{n(n+1)(2n+1)}{6}$$

b) Explain the principle of Inclusion - Exclusion.

c) State and prove Pigeonhole principle. Show that in a group of 13 children, there must be at least two children who were born in the same month. **(5+5+6)**

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4. a) Prove that, ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r} (0 \le r \le n)$

b) How many different numbers of 3 digits can be formed from the digits 1,2,3,4,5 and 6, if digits are not repeated? What will happen if repetitions are allowed?

c) Solve the following recurrence relation y_{n+2}-y_{n+1}-$2y_{n}$=n^2. **(4+6+6)**

5. a) Find the generating functions of the numeric functions, whose general terms are

i. $a^r = \left(\frac{r+2}{r}\right) \quad ; r \ge 0$ **(5+5+6)**

ii. $a^r = \left(\frac{4}{r}\right) \quad , r \ge 0$

b) Define equivalence relation. If R is the relation on $\mathbb{Z}$,the set of integers such that R={*(x,y)/ x-y* is divisible by 5}. Show that R is an equivalence relation.

c) Let R={(1,3),(4,5),(2,2)} and S={(1,2),(2,1),(3,3),(5,2)} be relations on the set A={1,2,....,5}. Find

i. RoS

ii. SoR

iii. Ro(SoR)

iv. SoSoS

6. a) If R^{-1} and S^{-1} are the inverses of the relations R and S respectively, then show that $(Ros)^{-1}=S^{-1}oR^{-1}$. **(5+5+6)**

b) Define a POSET. Let p={2,3,6,12,24,36}. Then draw the Hasse diagram of the poset $(p, \le)$ where $x \le y$ if x divides y.

c) Prove that the product of two posets is also a poset.

7. a) If $\{L, \leq\}$ is a Lattice, then for any a,b,c$\in$L, then prove that the following properties hold good. If $b \leq c$ then **(5+6+5)**

i. $a \vee b \leq a \vee c$

ii. $a \wedge b \leq a \wedge c$

b) Define distributive lattice. In a distributive lattice $\{L, \vee, \wedge\}$ if an element $a \in L$ has a complement, then prove that it is unique.

c) Prove that every distributive lattice is modular.

8. a) If $(L, *, \oplus)$ is distributive lattice and if $a * b = a * c$ and $a \oplus b = a \oplus c$ for all a,b,c$\in$L show that b=c. **(5+6+5)**

b) Show that if L is a distributive lattice then for all a,b,c$\in$L.

$(a * b) \oplus (b * c) \oplus (c * a) = (a \oplus b) * (b \oplus c) * (c \oplus a)$.

c) Show that D_{30} is a finite Boolean Algebra under partial order of divisibility.
