MMDSC-2.5

Sl.No. : **Total No. of Pages : 3**

II Semester M.Sc. Degree Examination, May/June - 2023
(CBCS, SLM Scheme)
MATHEMATICS
Discrete Mathematics

Time : 3 Hours **Max. Marks : 80**

Instructions: ***1) Answer any five full questions.***
2) All questions carry equal marks.

Q1) a) Define tuntology, contradiction and contingency statements. Show whether the following statements are tantology, Contradiction of contingency.

i) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

ii) $p \wedge (\sim p \wedge q)$

iii) $[(p \wedge q) \vee \sim r] \leftrightarrow p$

b) Define principle Disjunctive Normal form (PDNF) Principle conjunctive normal form (PCNF)

i) Obtain PDNF of $(p \wedge q) \vee (\sim p \wedge r) \vee (q \wedge r)$

ii) Obtain PCNF of $(\sim p \to r) \wedge (q \leftrightarrow p)$

[8+8]

Q2) a) i) Write a brief note on quantifier, universal quantifiers and existential quantifiers.

ii) Define negation of quantified statement. Then negate and simplify the following quantified statement

1) $\exists x, [p(x) \vee q(x)]$

2) $\forall x, [p(x) \to \sim q(x)]$

b) Establish the following results by using the principles of mathematical induction.

i) $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$

ii) $1^3 + 2^3 + 3^3 + ... + n^3 = \frac{n^2(n+1)^2}{4}$

[8+8]

Q3) a) State and prove pigeonhole principle and extended pigeonhole principle. Then prove by using extended pigeonhole principle that in any set of 29 persons at least five persons must have been born on the same day of the week.

b) i) Explain restricted permutation.

ii) If 10 items are present and 4 items are taken at a time then find the permutations of the following cases

a) Two things always occurs

b) Never occus

[10+6]

Q4) a) If ${}^{1000}C_{98} = {}^{999}C_{97} + {}^{x}C_{901}$. Find x.

b) Define recurrence relation. Show that the sequence {2,3,4,5....., 2+n,...} for $n \geq 0$ satisfies the recurrence relation $a_k = 2a_{k-1} - a_k - 2$, $k \geq 2$.

c) Determine the numeric function corresponding to each of the following generating functions

i) $\frac{1}{5 - 6x + x^2}$

ii) $\frac{(1+x)^2}{(1-x)^4}$

[5+5+6]

Q5) a) Explain all the properties of relation.
If R={(1,1), (1,2), (2,1), (2,2), (3,3), (3,4), (4,3), (4,4), (5,5)} is a relation on A={1,2,3,4,5}. Determine the properties of R.

b) Define composition of relations. If
R={(1,2), (4,5), (2,2), (3,1)} and S={(1,1), (2,1), (3,3), (5,3)} be relations on the set A={1,2,3,4,5} Find
   i) (R.S).S
   ii) R.R.R
   iii) S.S.S

[8+8]

Q6) a) Explain representation of relations using suitable examples.
b) If A,B, C and D are four non-empty sets, And R,S and T be three relations from A to B, B to C. and C to D respectively. Then Prove that (R.S).T=R.(SoT)
c) If R^{-1} and S^{-1} are the inverses of the relations R and S respectively. Then prove that $(R.S)^{-1}=S^{-1}.R^{-1}$

[6+5+5]

Q7) a) Define the following :
   i) POSET
   ii) Totally ordered set
   iii) Well-ordered set
   iv) Lexicographic ordering.
b) Consider the Set A={4,5,6,7}. Let R be the relation $\leq$ on A .Draw the directed graph and the Hasse diagram of R.

[8+8]

Q8) a) Define a lettice. Prove the following properties of lattice.
   i) If $\{L,\leq\}$ is a lattice, then for any $a,b,c\in L$ prove that,
   $a\wedge(b\vee c)\geq(a\wedge b)\vee(a\wedge c)$
   ii) If $\{L,\leq\}$ is a lattices then show that for any $a,b,c\in L$, $a\leq c$ if and only if $[a\vee(b\wedge C)\leq(a\vee b)\wedge C]$
b) Define distributive lattice and modular lattice and prove that every distributive lattice is modular.
c) If $(L,\vee,\wedge)$ is a complemented distributive lattice then, prove that
   i) $(a\vee b)'=a'\wedge b'$
   ii) $(a\wedge b)'=a'\vee b'$ for all $a,b\in L$.

[6+5+5]