MMDSC-2.5

Sl.No. :

Total No. of Pages : 3

II Semester M.Sc. Degree Examination, May - 2024

(CBCS, SLM Scheme)

MATHEMATICS

Discrete Mathematics

Time : 3 Hours **Max. Marks : 80**

Instructions: ***1) Answer any five full questions.***

2) All quesstions carry equal marks.

Q1) a) Explain conditional and biconditional statements. Construct the truth table for the following statements:-

i) $\sim (p \vee q) \vee (\sim p \wedge \sim q)$

ii) $(p \rightarrow r) \wedge (q \rightarrow r)$

b) Define tautology, contradiction. Identify whether given statements are tautology or contradiction.

i) $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

ii) $\sim (q \rightarrow r) \wedge r \wedge (p \rightarrow q)$.

[8+8]

Q2) a) Prove that $\sqrt{2}$ is irrational using the method of contradiction.

b) Define disjunctive normal form and conjunctive normal form. And obtain principle disjunctive normal form of $(\sim p \vee \sim q) \rightarrow (\sim p \wedge q)$.

c) Establish the following result by using mathematical induction.

$1^2+3^2+5^2+...+(2n-1)^2 = \dfrac{n(4n^2-1)}{3}$.

[4+8+4]

Q3) a) State and prove principle of Inclusion - Exclusion. Find how many positive integers not exceeding 500 are divisible by 7 or 11.

b) State and prove pigeonhole principle and extended pigeonhole principle. If 9 books are to be kept in 4 shelves. Then prove that, there must be at least one shelf which will contain at least 3 books.

[8+8]

Q4) a) Write the difference between a permutation and combination.

b) Prove the following results:

i) ${}^{n-1}C_r = {}^{n}C_r + {}^{n}C_{r-1}$

ii) $C(n, n) = C(n, 0) = 1$

c) If six papers are set in an examination of which two are mathematics. Then in how many different orders can the papers be arranged so that

i) the two mathematics papers are together.

ii) the two mathematics papers are not consecutive

[4+6+6]

Q5) a) Find the first four terms of the following recurrence relations.

i) $a_k = 2a_{k-1} + k$, for all integers $k \geq 2$, $a_1 = 1$

ii) $a_k = a_{k-1} + 3a_{k-2}$, for all integers $k \geq 2$, $a_0 = 1$, $a_2 = 2$.

b) Solve the recurrence relation of the Fibonacci sequence of numbers $f_n = f_{n-1} + f_{n-2}$, $n \geq 2$ with the initial condition $f_0 = 1$, $f_1 = 1$.

c) Solve the recurrence relation $a_{n-2} - 5a_{n-1} + 6a_n = 2$ with initial condition $a_0 = 1$ and $a_1 = -1$.

[4+6+6]

Q6) a) Find the generating function of the following.

i) 1, 0, 1, 0, ...

ii) 2, –2, 2, –2, ...

b) Define equivalence relation. If R is the relation on $\mathbb{Z}$, the set of integers such that $R = \{(x, y)/x - y \text{ is divisible by } 5\}$, show that R is an equivalence relation.

c) Let A = {1, 2, 3, 4}, B = {p, q, r, s} and C = {x, y, z} and also R = {(1, p), (1, r), (1, s), (2, r), (2, s), (3, p), (4, s)} and S = {(p, x), (p, z), (q, y), (r, y), (r, z), (s, x), (s, z)} find the composition of R and S.

[6+6+4]

Q7) a) Explain in brief the representation of relations using suitable examples.

b) Define POSET and linearly ordered set. Show that the relation '<' (less than) defined on N, the set of positive integers is neither an equivalence relation nor partially ordered relation but it is a totally ordered relation.

c) Let P = {2, 3, 4, 6, 12, 36, 48} be a non-empty set. Then draw the Hasse diagram of the poset $(P, \leq)$ where $x \leq y$ if x divides y.

[6+6+4]

Q8) a) If $\{L, \leq\}$ is a lattice, then prove that for any $a, b, c \in L$, $a \leq c$. if and only if $a \vee (b \wedge c) \leq (a \vee b) \wedge c$.

b) Define sublattice. And determine all the sub-lattice of D_{30} that contain at least four elements.

c) Prove the following results:

i) In a distributive lattice $\{L, \vee, \wedge\}$ if an element $a \in L$ has a complement, then prove that it is unique.

ii) If $(L, *, \oplus)$ is distributive lattice and if $a * b = a * c$ and $a \oplus b = a \oplus c$ for all $a, b, c \in L$, show that $b = c$.

[4+4+8]

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