MMDSC-2.5

Sl.No. :

Total No. of Pages : 3

Second Semester M.Sc. Degree Examination, November - 2024
(SLM Scheme) (CBCS)
MATHEMATICS
Discrete Mathematics

Time : 3 Hours **Max. Marks : 80**

Instructions : 1) Answer any five full questions.
2) All questions carry equal marks.

Q1) a) Define proposition and logical connections. Construct the truth tables for all the logical connectives.

b) i) Define tautology, contradiction and contingency statements verify whether the given compound proposition $(\sim(p\wedge q)\vee r)\Rightarrow\sim P$ is tautology or not.

ii) Write the converse, inverse and contrapositive of the statement. "If its minus degree celsius then it's very cold".

[8 + 8]

Q2) a) Prove the following statement by indirect proof "If $x+y$ is odd and x is even, then y is odd".

b) Prove the following logical equivalences without using the truth table.
$p\rightarrow(q\rightarrow r)\equiv(p\wedge q)\rightarrow r$.

c) Define principle Disjunctive normal form (PDNF) and principle conjunctive normal form (PCNF).

i) Obtain the PDNF of $(\sim p\vee\sim q)\rightarrow(\sim p\wedge r)$.

ii) Obtain the PCNF of $(p\wedge q)\vee(\sim p\wedge r)$

[4 + 4 + 8]

Q3) a) Using mathematical induction method, prove that $2n+1\leq 2^m$ for $n\geq 3$.

b) State and prove principle of inclusion and exclusion.

c) State and prove the pigeon hole principle.

[5 + 5 + 6]

Q4) a) Prove the following :

i) ${}^{n}C_{r} + {}^{n}C_{r-1} = {}^{n+1}C_{r}\ (0 \le r \le n)$

ii) ${}^{n-1}C_{r} = {}^{n}C_{r} + {}^{n}C_{r}$

b) How many different words can be made out of the letters of the word 'ALLAHABAD'? In how many of these with the vowels occupy the even places?

c) If ${}^{1000}C_{98} = {}^{999}C_{97} + {}^{x}C_{901}$. Find x

[8 + 5 + 3]

Q5) a) Solve the recurrence relation of the Fibonacci sequence of numbers $f_n = f_{n-1} + f_{n-2}$, $n \geq 2$ with the initial condition $f_0 = 1, f_1 = 1$.

b) Solve the recurrence relation, $a_{n+2} - 4a_{n+1} + 4a_n = 2n$.

c) Show that the time complexity of tower of Hanoi problem is $T(n) = O(2^n)$.

[6 + 6 + 4]

Q6) a) Determine the numeric function corresponding to the given generating functions

i) $\dfrac{1}{5 - 6x + x^2}$

ii) $\dfrac{(1+x)^2}{(1-x)^4}$

b) If R is the relation on Z, the set of integers such that $R=\{(x,y)/x-y$ is divisible by $5\}$, show that R is an equivalence relation.

c) Define composite relation. If R and S are two relations on the set A={1,2,....,5} Given R={(1,3), (4,5), (2,2)} and S={(1,2),(2,1),(3,3),(5,2)} be relations on the set A. Find Ro(SoR),(RoS)oR,SoSoS and RoRoR.

[6 + 4 + 6]

Q7) a) If R^{-1} and S^{-1} are the inverses of the relations R and S respectively, then prove that, $(RoS)^{-1} = S^{-1} oR^{-1}$.

b) Define totally ordered set. Show that the relation '<' (less than) defined on N, the set of positive integers is neither an equivalence relation nor partially ordered relation but is a totally ordered relation.

c) For a set A={4,5,6,7}. Let R be the relation $\leq$ on A. Draw the directed graph and Hasse diagram of R.

[5 + 5 +6]

Q8) a) Define minimal and maximal element of a POSET. Find the minimal and maximal elements of the POSET $(P,\leq)$, Where P={1,2,3,4,6,8,12} and $\leq$ is the relation of divisibility.

b) State the properties of bounded lattices prove that, every finite lattice $L=\{a_1,a_2,..,a_n\}$ is bounded.

c) Prove the following :

i) Every chain is a distributive lattice

ii) In a distributive lattice, if an element has a complement then this complement is unique.

[4 + 4 + 8]

