MMDSC-2.5

Sl.No. :

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Second Semester M.Sc. Degree Examination, October - 2025
(SLM Scheme) (CBCS)
MATHEMATICS
Discrete Mathematics

Time : 3 Hours **Max. Marks : 80**

Instructions : 1) Answer any five full questions.
2) All questions carry equal marks.

Q1) a) Define proposition and logical connectives. Construct the truth table for all the logical connectives.

b) State and prove the following laws using truth table.
i) Distributive laws
ii) De' morgans laws

[8 + 8]

Q2) a) Test whether the following arguments are valid.
i) $(\neg p \leftrightarrow q) \wedge (q \rightarrow r) \wedge \neg r / \therefore p$
ii) $(p \rightarrow q) \wedge (r \rightarrow s) \wedge (p \vee r) / \therefore (q \vee s)$

b) Define Principle Disjunctive Normal Form (PDNF) and Principle Conjunctive Normal Form (PCNF)
i) Obtain PDNF of $(\neg p \vee \neg q) \rightarrow (\neg p \wedge r)$
ii) Obtain PCNF of $(p \wedge q) \vee (\neg p \wedge r)$

[8 + 8]

Q3) a) Prove by contradiction that if $x + y > 5$ then either $x > 2$ or $y > 3$.

b) Using mathematical induction method, prove that $(2n + 1) \leq 2^m$ for $n \geq 3$.

c) State and prove the extended pigeonhole principle.

[5 + 5 + 6]

Q4) a) If ${}^{1000}C_{98} = {}^{999}C_{97} + {}^{x}C_{901}$. Find x.

b) Define recurrence relation. Show that the sequence $\{2, 3, 4, 5, ..., 2 + n, ...\}$ for $n \geq 0$ satisfies the recurrence relation $a_k = 2a_{k-1} - a_k - 2$, $k \geq 2$

c) Solve $a_n = a_{n-1} + 2a_{n-2}$, $n \geq 2$ with the initial conditions $a_0 = 0$, $a_1 = 1$.

[5 + 6 + 5]

Q5) a) Explain all the properties of a relation. If R = {(1,1), (1,2), (2,1), (2,2), (3,3), (3,4), (4,3), (4,4), (5,5),} is a relation on A = {1,2,3,4,5}. Determine the properties of R.

b) i) Let A,B,C and D be four non-empty sets Let R,S and T be the three relations from A to B, B to C and C to D respectively. Then prove that $(R \circ S) \circ T = R \circ (S \circ T)$.

ii) If R^{-1} and S^{-1} are the inverse of the relation R and S respectively, then prove that $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$

c) Define POSET, totally ordered set, well ordered set and lexicographic ordering, write suitable example for each of these sets.

[5 + 6 + 5]

Q6) a) Show that the relation '<' (less than) defined on N, the set of positive integers is neither an equivalence relation nor partially ordered relation but is a totally ordered relation.

b) Define dual of a POSET and prove that the product of two posets is also a poset.

c) Let P = {2, 3, 6, 12, 24, 36} then draw the Hasse diagram of the poset (P, ≤) where $x \leq y$ if x divides y.

[6 + 5 + 5]

Q7) a) Let (P, ≤), be a poset. Then prove that P has at least one maximal element and at least one minimal element.

b) Let P = {1, 2, 3, 4, 6, 12}. Then prove that the posets (P,1) and (P, ≤) where ≤ denotes the usual "less then or equal to" are not isomorphic.

c) Prove that every finite lattice L = $\{a_1, a_2, a_3, ..., a_n\}$ is bounded.

[6 + 6 + 4]

Q8) a) If {L, ≤} is a lattice, then for any $a, b, c \in L$ Prove the following properties of lattice.

i) Idempotent law

ii) Commutative law

iii) Associative law

iv) Absorption law

b) Prove the following :-

i) In a distributive lattice {L, ∨, ∧} if an element $a \in L$ has a complement, then prove that it is unique.

ii) If {L, ∨, ∧} is a complemented distributive lattice then prove that $(a \vee b)' = a' \wedge b'$ and $(a \wedge b)' = a' \vee b'$ holds for all $a, b \in L$.

[8 + 8]

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