MMDSC-3.1

Sl.No. : **Total No. of Pages : 3**

III Semester M.Sc. Degree Examination, February - 2024
(SLM Scheme CBCS)
MATHEMATICS
Course-I : Linear Algebra

Time : 3 Hours **Max. Marks : 80**

Instructions : ***1) Answer any five full questions.***
2) All questions carry equal marks.

Q1) a) Define a subspace. Let $\mathbb{R}$ be the field of real numbers and S be the set of all solutions of the equations $x + y + 2z = 0$ Then, show that S is a subspace of $\mathbb{R}^3$.

b) Express $v = \{1, -2, 5\}$ in $\mathbb{R}^3$ as a linear combination of the following vectors: $v_1 = (1,1,1)$, $v_2 = (1,2,3)$, $v_3 = (2,-1,1)$.

c) Define a linear transformation. If the mapping $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x, y) = (4x + 5y, 6x - y)$, then show that T is linear transformation.

[6+6+4]

Q2) a) State and prove the dimension theorem for linear transformation.

b) Define algebra of linear transformation. Explain the properties of algebra of linear transformation.

c) Show that product of two linear transformations is a linear transformation.

[7+5+4]

Q3) a) If V is finite dimensional vector space over a field F, then show That $T \in A(V)$ is non-invertible (Singular) if and only if there exists a $v \neq 0$ in V such that $T(v) = 0$.

b) Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation by $T(x, y) = (2x + y, 3x + 2y)$. Then show that T is invertible and find T^{-1}.

c) Define an elementary row operation. Explain the three types of operation with on example.

[5+5+6]

Q4) a) Relative to the basis $B = \{v_1, v_2\} = \{(1,1),(2,3)\}$ of $\mathbb{R}^2$ find the coordinate matrix of.

i) $v = (4,-3)$

ii) $v = (a,b)$

b) If a matrix $A = [aij]_n$ is upper (lower) triangular, then show that det $(A) = a_{11}a_{22}.....a_{nn}$. Further wirte the complete expression for det (A) if $A = [aij]_3$

c) Explain the difference between homogeneous and non-homogeneous system of equation with an example.

[6+6+4]

Q5) a) Define Minors and cofactors. Explain the Cremmer's rule with an example.

b) Prove that a square matrix A has O as eigen values if and only if A is not invertible.

c) Let V be an inner product space over F. Then Prove the following:

i) $|< x, y >| = || x || \ || y ||$

ii) $|| x + y || \leq || x || + || y ||$ for all $x, y \in V$

[7+4+5]

Q6) a) Apply the Gram-Schmidt orthogonalization process to the basis $B = \{(1,0,1),(1,0,-1),(0,3,4)\}$ of the inner product space $\mathbb{R}^3$ to find an orthogonal and an orthonormal basis of $\mathbb{R}^3$.

b) State and prove the Bessel's inequality.

c) Find the adjoint of linear transformation $T:\mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x, y) = (x + 2y, x - y) \ \forall (x,y) \in \mathbb{R}^2$.

[6+6+4]

Q7) a) Show that the linear map satisfying $T^2 = T$ is diagonalizable.

b) If the matrix $A \in F_n$ has all its eigen values in F, then show that there is a matrix $C \in F_n$ such that CAC^{-1} is a triangular matrix.

c) If V is n-dimensional vector space over a field F and $T \in A(V)$ has all its eigen values in F, then show that T satisfies a polynomial of degree *n* over F.

[4+6+6]

Q8) a) Show that the linear transformation T is diagonalizable if and only if its minimal polynomial is a product of distinct linear factors.

b) Define Jordan canonical form with an example. Determine the Jordan cononical form of A.

Where, $A = \begin{bmatrix} 2 & 6 & -15 \\ 1 & 1 & -5 \\ 1 & 2 & -6 \end{bmatrix}$

c) Find all possible rational canonical forms for 6×6 matrices with $(x-2)(x+2)^3$ as minimal polynomial.

[5+5+6]

