MMDSC-3.1

Sl.No. : **Total No. of Pages : 3**

III Semester M.Sc. Degree Examination, June/July - 2024
(CBCS, SLM Scheme)
MATHEMATICS
Course-I : Linear Algebra

Time : 3 Hours **Max. Marks : 80**

Instructions : ***1) Answer any five full questions.***
2) All questions carry equal marks.

Q1) a) Define a vector space with on example. Explain the general properties of vector space.

b) Verity whether the vectors (1,–2, 2), (3,0,4), (–2, 2, –4) are linearly independent in $V_3(R)$

c) Show that the span of any subset S of a vector space V is a subspace of V. further, show that any subspace of V that contains S must also contain the span of S.

[6+5+5]

Q2) a) Define a linear transformation Let U and V be vector space over the same filed F. Then show that the set of all linear. transformation of U into V is a vector space over F.

b) Let T be a linear transformation of U into V. If U has finite dimension, then prove that rank (T)+ nullity (T)=din (U).

c) If V is finite dimensional vector space over a field F, then show that $T \in A(V)$ is non-invertible (Singular) if and only if there exists $av \neq o$ in V such that $T(v) = 0$.

[6+6+4]

Q3) a) If A is an algebra, with unit element, over F, then show that A is isomorphic to a sub algebra of A (V) for some vector space V over F.

b) Define an augmented matrix and an invertible matrix. Determine whether the matrix. $A=\begin{bmatrix}0 & 2 & 4\\ 2 & 4 & 2\\ 3 & 3 & 1\end{bmatrix}$ is invertible of not.

c) Show that for any matrix A=[a,j],

$\det(A)=\sum_{(j)}(-1)^t a_{1j_1}a_{2j_2}.....a_{nj_n}$, where $\sum_{(i)}$ denotes the sum over all possible permutations $i_1, i_2,.....i_n$ of 1, 2, ..., *n*, and S is the number of interchanges used to carry $i_1, i_2,.....,i_n$ into the natural ordering.

[5+3+8]

Q4) a) Define Minors and cofactors. Explain the crammer's rule with an example.

b) State and prove cayley-Hamilton theorem.

[8+8]

Q5) a) Let T be a linear operator on a vector space V, and $\lambda_1,\lambda_2,.....,\lambda_k$ be distinct eigen values of T. If $\nu_1,\nu_2,....,\nu_k$ age eigenvectors of T such that λ_i. corresponds to ν_i $(1\le i\le k)$, then that $\{\nu_1,\nu_2,...,\nu_k\}$ is linearly independent.

b) Define an inner product space and establish triangular inequality.

c) Define an orthogonal and orthonormal set of vectors. If $\{\nu_1,\nu_2,...,\nu_k\}$ is an orthonormal set in an inner product space. V, and $\nu\in V$. Then prove that

$\sum_{i=1}^{k}\left|\langle\nu,\nu_i\rangle\right|^2\le\|V\|^2$ **[5+6+5]**

Q6) a) Define a normal operator. Prove that an operator T is normal if and only if there exists an orthonormal basis of V, a finite dimensional inner product space consisting of eigenvectors of T.

b) Let $\omega_1 = (1,0,1,0), \omega_2 = (1,1,1,1) and\ \omega_3 = (0,1,2,1)$ in $\mathbb{R}^4$. If $\{\omega_1, \omega_2, \omega_3\}$ is linearly independent. Use the Gram-Schmidt orthogonalization process to compute the orthogonal vectors ν_1, ν_2 and ν_3 and then find the normalize these vectors to obtain an orthonormal set.

c) Let V be a finite dimensinal inner product space and let T be a linear operator on V. then show that there exists a unique function $T^*: V \to V$ such that $(T(x), y) = (x, T^*(y))$ for all $x, y \in V$ furthermore, show that T*is linear

[5+6+5]

Q7) a) Test the matrix $A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} \in M_{3\times3}(R)$ diagonalizable of not.

b) If T and U are simultaneously diagonalizable operators, then show that T and U commute.

c) If $T \in A(V)$ has all its characteristic roots in F, then show that there is a basis of V in which the matrix of T is triangular.

[5+5+6]

Q8) a) Find all possible Jordan forms A for 6×6 matrical with $x^2(1–x^2)$ as minimal polynomical.

b) If $W \subset V$ is invariant under T, then T induces a linera transformation $\bar{T}$ on V/W, defined by $\bar{T}(V+W) = T(\nu) + W$. If T satisfies the polynomial $q(x) \in F[x]$. Then so does $\bar{T}$. if $p,(x)$ is the minimal polynomical for $\bar{T}$ over F and if $P_1(x)$ is the for T, the $P_1(x)/P(x)$

c) Show that the characteristic polynomial of the companion matrix $((q(x))$ is $q\ (x)$

[5+6+5]

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