MMDSC-3.1

Sl.No. :

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III Semester M.Sc. Degree Examination, May/June - 2026
(SLM Scheme, CBCS)
MATHEMATICS
Course-I : Linear Algebra

Time : 3 Hours **Max. Marks : 80**

Instructions : ***1) Answer any five full questions.***
2) All questions carry equal marks.

Q1) a) Define subspace. Write all the criteria of a subspace. Let R be the field of real numbers and S be the set of all solutions of the equations $x+y+2z=0$. Show that S is a subspace of R^3.

b) Verify whether given set of vectors (0, 1, 0, 1, 1), (0, 1, 0, 1, 1), (0, 1, 0, 1, 1) and (0, 1, 0, 1, 1) are linearly independent R^5.

c) Define maximal linearly independent subset. Let S be a linearly independent subset of a vector space V. Then prove that there exists a maximal linearly independent subset of V that contains S.

[6+5+5]

Q2) a) Let U and V be vector space over the same field F. Then prove that the set of all linear transformation of U into V is a vector space over F.

b) State and prove Dimension theorem.

c) Prove that the product of two linear transformations is a linear transformation.

[6+6+4]

Q3) a) If A is an algebra, with unit element, over F. Then prove that A is isomorphic to a subalgebra of A(V) for some vector space V over F.

b) Solve the system of equations

$$x_1 + 2x_2 + 2x_3 = 4$$
$$x_1 + 3x_2 + 3x_3 = 5$$
$$2x_1 + 6x_2 + 5x_3 = 6$$

c) If $A = \left[a_{ij} \right]_n$, then prove that $\det(A^T) = \det(A)$

[7+5+4]

Q4) a) Let $A \in M_{m\times n}(F)$ be a matrix. Then prove that a scalar λ is an eigen value of A if and only if $\det(A-\lambda I_n)=0$.

b) Find the eigen values of the given matrix.

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

c) State and prove Cayley-Hamilton theorem.

[4+4+8]

Q5) a) Let V be an inner product space over F. Then prove that for all $x, y \in V$ and $C \in F$, the following statements are true.

i) $\|Cx\| = |C| . \|x\|$

ii) $\|x\| = 0$ if and only $x = 0$. In any case $\|x\| \geq 0$

iii) Cauchy-Schwarz inequality

iv) Triangle inequality

b) State and prove Bessel's inequality

[8+8]

Q6) a) Let V be an inner product space and let T be a linear operator on V. Then prove that T is an orthogonal projection if and only if T has an adjoint T* and $T^2=T=T^*$.

b) Define symmetric bilinear from. Let B be a bilinear form on a finite-dimensional vector space V and let B be an ordered basis for a vector space V. Then prove that B is symmetric if and only if $\psi_B(B)$ is symmetric.

c) Find the rank and signature of the quadratic form $x_1^2 - 4x_1x_2 + x_2^2$.

[7+5+4]

Q7) a) If T and U are simultaneously diagonalizable operators, then show that T and U commute.

b) If $T \in A(V)$ has all its eigen values in F, then there is a basis of V in which the matrix of T is triangular.

c) If the matrix $A \in Fn$ has all its eigen values in F, then prove that there is a matrix $C \in Fn$ such that CAC^{-1} is a triangular matrix.

[3+8+5]

Q8) a) Let $V = V_1 \oplus V_2$, where V_1 and V_2 are subspaces of V invariant under T. Let T_1 and T_2 be the linear transformations induced by T on V_1 and V_2 respectively of T_2 is $P_2(x)$. Then prove that the minimal polynomial for T over F is the least common multiple of $P_1(x)$ and $P_2(x)$.

b) Let $P(x)$ be a minimal polynomial of a linear operator T on a finite dimensional vector space V. Then prove that,

i) The minimal polynomial of T exists and unique.

ii) If $P(x)$ is the minimal polynomial of T and $h(x)$ is any polynomial with $h(T)=0$, then $p(x)$ is a factors of $h(x)$.

[8+8]

