MMDSC-3.1

Sl.No. :

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III Semester M.Sc. Degree Examination, June - 2025
(CBCS, SLM Scheme)
MATHEMATICS
Course - I : Linear Algebra

Time : 3 Hours **Max. Marks : 80**

Instruction : Answer any five full questions.

Q1) a) Let V be a vector space and S a subset that generates V. If A is a maximal linearly independent subset of S, then prove that A is a basis for V.

b) Express $V = \{1, -2, 5\}$ in R^3 as a linear combination of the following vectors $V_1 = (1, 1, 1)$, $V_2 = (1, 2, 3)$, $V_3 = (2, -1, 1)$.

c) Define linear transformation, Let T be a linear transformation of U into V. If W is any subspace of U, then prove that T(w) is a subspace of V.

[6+5+5]

Q2) a) If B results from matrix A by interchanging two rows (columns) of A, then prove that det(B) = –det(A).

b) State and prove Rank-Nullity theorem for linear transformation.

c) Determine whether the matrix is $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & -1 \\ 1 & 5 & 4 \end{bmatrix}$ invertible or not.

[6+5+5]

Q3) a) If λ is an eigenvalue of an invertible matrix A over a field F, then show that λ^{-1} is an eigenvalue of A^{-1}.

b) Let T be a linear operator on an *n*-dimensional vector space V. If T has *n* distinct eigenvalues, then prove that T is diagonalizable.

c) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 4 \end{bmatrix}$.

[5+5+6]

Q4) a) State and prove Cauchy-Hamilton theorem.

b) Define an inner product space and establish the triangle inequality.

c) Let V be a finite dimensional inner product space show that V has an orthonormal set as a basis.

[6+5+5]

Q5) a) Let T be a linear operator on a finite dimensional complex inner product space V, then prove that T is normal if and only if there exist an orthonormal basis for V consisting of eigenvector of T.

b) State and prove Spectral theorem.

c) Find the rank and signature of the quadratic form $X_1^2 - 4X_1X_2 + X_2^2$.

[5+6+5]

Q6) a) Let V be an inner product space and let T be a linear operator on V, then prove that T is an orthogonal projection if and only if T has an adjoint T^* and $T^2 = T = T^*$.

b) Let A be a complex $n \times n$ matrix, then prove that A is normal if and only if A is unitary equivalent to a diagonal matrix.

c) Define a normal operator. Prove that an operator T is normal if and only if there exists an orthonormal basis of V, a finite dimensional inner product space consisting of eigenvectors of T.

[5+5+6]

Q7) a) Test the matrix $A=\begin{bmatrix}3 & 1 & 0\\ 0 & 3 & 0\\ 0 & 0 & 4\end{bmatrix}\in M_{3\times 3}(R)$ diagonalizable or not.

b) Find the rational canonical form of $A=\begin{bmatrix}1 & 1 & 1 & 1\\ 0 & 0 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & -1 & 1 & 0\end{bmatrix}$.

c) Prove that the linear transformation T is diagonalizable if and only if its minimal polynomial is a product of distinct linear factors.

[6+5+5]

Q8) a) Reduce to Jordan canonical form the matrix $\begin{bmatrix}1 & 1 & 1\\ -1 & -1 & -1\\ 1 & 1 & 0\end{bmatrix}$.

b) Find the minimal polynomial of the matrix $\begin{bmatrix}0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0\end{bmatrix}$.

c) Reduce to quadratic form $2X_1X_2-3X_2X_3+4X_1X_3$ to diagonal form.

[6+5+5]

