**MMDSC-3.2**

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# III Semester M.Sc. Degree Examination, February - 2024
## (SLM Scheme CBCS)
## MATHEMATICS
## Course - II : Topology - I

**Time : 3 Hours** **Max. Marks : 80**

***Instruction :*** ***Answer any five full questions.***

***Q1)*** a) Define inverse function. Show that if $f$ ; A $\rightarrow$ B is bijection then $f^{-1}$; B$\rightarrow$A is also bijection.

b) Let $\beta$ and $\beta'$ be bases for topology $\tau$ and $\tau'$ respectively on X. Then prove the following are equivalent.

i) $\tau'$ is finer then $\tau$

ii) For each $x \in X$ and each basis element $B \in \beta$ containing $x$, there is an basis element $B' \in \beta'$ such that $x \in B' \subset B$.

c) Show that the collection $\beta = \{(a, b) \mid a < b, a.\ b \in \mathbb{R}\}$ the set of all open intervals in the real line $\mathbb{R}$.

**[6+6+4]**

***Q2)*** a) Define subbasis. Let X and Y be two topological spaces then prove the collection $\tau = \{\Pi,^{-1}(U) \mid U \text{ is open in } X\} \cup \{\Pi^{-1},(V) \mid V \text{ is open in } Y\}$ is a subbasis for the product topology on X × Y.

b) Define product topology. If A is a subspace of X and B is a subspace of Y then prove that the product topology on A × B the same as the topology A × B inherits as the subspace of X × Y.

**[8+8]**

***Q3)*** a) Define closed set show that every finite point set in a Hausdorff space is closed.

b) P.T. the product of two Hausdorff spaces is Hausdorff.

c) Let $A_\alpha$ be a subspace of $X_\alpha$ for each $\alpha \in J$ then prove that $\prod_{\alpha \in J} A_\alpha$ is a subspace of $\prod_{\alpha \in J} X_\alpha$ if both the products are given the box topology.

**[4+6+6]**

***Q4)*** a) Show that the topologies on $\mathbb{R}^n$ induced by the Euclidian metric d and the square metric $\rho$ are the same as the product topology on $\mathbb{R}^n$.

b) State and prove uniform limit theorem.

**[8+8]**

***Q5)*** a) P.T. a finite Hausdorff space is totally disconnected

b) The usual topological space $(\mathbb{R}.\mathrm{U})$ is connected.

c) Define path connected. Show that the continuous image of a path connected space is path - connected.

**[6+6+4]**

***Q6)*** a) Show that the relation "$x \sim y$" is an equivalence relation.

b) Define locally connected. Prove that X is locally connected if and only if there is a basis for X consisting of connected sets.

c) Prove that the subspace $[-1, 0) \cup (0, 1]$ of $\mathbb{R}$ is not connected but it is locally connected.

**[6+7+3]**

***Q7)*** a) Show that every compact subset of a Hausdorff space is closed.

b) State and prove Heine - Borel theorem.

**[8+8]**

***Q8)*** a) Let *x* be a metrizable space then prove the following are equivalent.

i) X is compact

ii) X is limit point compact

iii) X is sequentially compact

b) Show that every closed subset of a locally compact space is locally compact.

**[10+6]**

