MMDSC-3.2

Sl.No. : **Total No. of Pages : 2**

III Semester M.Sc. Degree Examination, June/July - 2024
(SLM Scheme, CBCS)
MATHEMATICS
Course-II : Topology-I

Time : 3 Hours **Max. Marks : 80**

Instruction : ***Answer Any five full questions.***

Q1) a) Define finite and infinite set show that only one-one onto mapping has an inurse mapping.

b) Let X be a set and let β be a basis for a topology τ on X. then P.T. every $U \in \tau$ equals the collection of the union of elements of β

c) Define sub basis then show that collection β of all finite intersections of the element of τ form a basis for a topology on X.

[5+5+6]

Q2) a) Define product topology on X ×Y. If β is a basis for the topology of X and $\mathcal{C}$ is a basis for the topology of Y. then P.T. the collection $\mathcal{D}=\{B\times C | B\in \beta \text{ and } C\in \mathcal{C}\}$

b) Define subspace topology. If β is a basis for a topological space of X. Then prove the collection $\beta_y = \{B \cap Y \mid B \in \beta\}$ is a basis for the subspace topology of Y.

[8+8]

Q3) a) Define subset of X. Let A be a subset of the topological space X then prove that $\overline{A} = A \cup A'$

b) Let $f : A \to \prod_{\alpha \in J} X_\alpha$ be given by the equation $f(a) = (f_\alpha(a))_{\alpha \in J}$, where $f_\alpha : A \to x_\alpha$, for each α. Let $\prod_{\alpha \in J} X_\alpha$ have the product topology then Prove that the function *f* is continuous if and only if each function f_α is continuous.

c) Give the comparision of the box and product topologies.

[5+6+5]

Q4) a) Let d and d' be two metric on the set X. Let τ and τ' be the topologies they induce respectively than show that τ' is finer than τ iff for each $x \in x$ and each $\in > 0.$ there exist a $\delta > 0$ such that $B_d(x,\delta) \subset B_d(x,\in)$

b) State and prove uniform limit theorem.

[8+8]

Q5) a) Show that the cartesian product of connected spaces is conected.

b) State and prove intermediate value theorem.

c) Define path connected. P.T.A. path-Connected space is connected.

[6+5+5]

Q6) a) Define component of a space. Show that every component of a topological space is a closed set.

b) Define locally connected. Prove that every component of a locally connected space is open.

c) If X is a topological space, each path component of X lies in a component of X. If is X is locally path connected then P.T. the components and the path component of X are the same.

[5+5+6]

Q7) a) Let $A \subseteq Y$ and (y,τ^*) be a subspace of (x,τ) then P.T. set A is T* compact if and only if it is τ compact.

b) State and prove maximum and minimum value theorem.

c) Show that every closed and bounded subspace of $\mathbb{R}^n$ is compact

[5+5+6]

Q8) a) State and prove uniform continuity theorem.

b) Show that every closed subset of a locally compact space is locally compact.

[8+8]

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