MMDSC-3.2

Sl.No. : **Total No. of Pages : 2**

III Semester M.Sc. Degree Examination, May/June - 2026
(SLM Scheme, CBCS)
MATHEMATICS
Course - II : Topology - I

Time : 3 Hours **Max. Marks : 80**

Instruction : Attempt any five full questions.

Q1) a) Let β and β' be bases for topology τ and τ' respectively on X. Then prove the following are equivalent.
i) τ' is finer than τ.
ii) For each $x \in X$ and each basis element $B \in \beta$ containing X, there is a basis element $B' \in \beta'$ such that $x \in B' \subset B$.

b) Define discrete and indiscrete topological spaces. Let X = {1, 2, 3} and τ = {ϕ, X, {1}, {2}, {1, 2}}. Show that τ is a topology on X.

c) Let $\mathscr{B}$ be a basis for a topology on X. Let τ be defined by $\tau = \{U \subset X |$ for each $x \in U$ there is an element $B \in \mathscr{B}$ such that $x \in B \subset U$. Then show that τ is a topology on X.

[6+4+6]

Q2) a) Define product topology. Let X and Y be two topological spaces, the show that the collection S = $\{\pi_1^{-1}(U)/U$ is open in X$\}$ U$\{\pi_1^{-1}(V)/V$ is open in Y$\}$ is a subbasis for the product topology on X × Y.

b) If $\mathscr{B}$ is a basis for the topology of X and $\mathscr{C}$ is a basis for the topology of Y. Then show that the collection $\mathscr{D}$ = {B×C|B$\in\mathscr{B}$ and C$\in\mathscr{C}$} is a basis for the topology of X × Y.

[8+8]

Q3) a) Let A be a subset of the topological space X, then show that $\bar{A} = A \cup A'$.

b) Prove that the product of two Hausdorff spaces is Hausdorff.

c) Let X be a topological space. Then show that the following conditions holds.
i) ϕ and X are closed.
ii) Arbitrary intersections of closed sets are closed.
iii) Finite unions of closed sets are closed.

[5+6+5]

Q4) a) Let d be metric on X. Suppose $y \in B_d(x, \varepsilon)$. Then show that there exists a $\delta > 0$. such that δ-ball centered at y is contained in $B_d(x, \varepsilon)$.

b) Show that the topologies on $\mathbb{R}^n$ induced by the Euclidean metric d and the square metric δ are the same as the product topology on $\mathbb{R}^n$.

c) Let $p : X \to Y$ be a quotient map and let Z be a space and let $p : X \to Z$ be a continuous map that is constant on each set $p^{-1}(y)$, for $y \in Y$. Then show that g induces a continuous map $f : Y \to Z$ such that, $f \circ p = g$.

[5+6+5]

Q5) a) Let Y be the subspace of X, is connected then show that there exists no separation of Y.

b) Show that the usual topological space $(\mathbb{R}, U)$ is connected.

c) Define path connected. Show that the continuous image of a path connected space is path connected.

[4+8+4]

Q6) a) Define locally connected set. Prove that X is locally connected if and only if there is a basis for X conserting of connected sets.

b) Prove that a topological space X is locally path connected if and only if for every open set U of X, each component of U is open in X.

c) Prove that the subspace $[-1, 0) \cup (0, 1]$ of $\mathbb{R}$ is not connected but it is locally connected.

[6+6+4]

Q7) a) State and prove Heine Borel theorem.

b) Show that the image of compact space under a continuous map is compact.

c) Show that every compact topological space is countable compact.

[8+5+3]

Q8) a) Let X be a metrizable space then prove the following are equivalent.

i) X is compact

ii) X is limit point compact

iii) X is sequentially compact

b) Show that every closed and bounded subspace of $\mathbb{R}^n$ is compact.

[10+6]

