MMDSC-3.2

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III Semester M.Sc. Degree Examination, June - 2025
(CBCS, SLM Scheme)
MATHEMATICS
Course - II : Topology - I

Time : 3 Hours **Max. Marks : 80**

Instruction : Answer any five full questions.

Q1) a) Define inverse function. Show that only one-one onto mapping has an inverse mapping.

b) Define indiscrete and discrete topology. Let $X=\{a,b,c\}$ and $\tau=\{\phi, X, \{a\}, \{a,b\}, \{a,c\}\}$. Show that τ is a topology on X.

c) Let X be a topological space. Suppose that $\mathscr{C}$ is a collection of open sets of X such that for each $x \in X$ and each open set of U of X containing x, there is an element C of $\mathscr{C}$ such that $x \in C \subset U$. Then show that $\mathscr{C}$ is a basis for the topology on X. Further, a topology generated by $\mathscr{C}$ is same as any other topology on X.

[4+4+8]

Q2) a) Define subbasis topology. Let $\mathscr{S}$ be a subbasis then prove the collection β of all finite intersection of the element of $\mathscr{S}$ form a basis for a topology on X.

b) Define product topology. Let X and Y be two topological spaces, then show that the collection $\mathscr{S}=\{\pi_1^{-1}(U)|U \text{ is open in } X\} \cup \{\pi_1^{-1}(V)|V \text{ is open in } Y\}$ is a subbasis for the product topology on X × Y.

c) Define subspace topology. If β is a basis for a topological space of X. Then show that the collection $\beta_Y=\{B \cap Y | B \in \beta\}$ is a basis for the subspace topology of Y.

[5+6+5]

Q3) a) Let A be a subset of the topological space X. Then show that $\bar{A} = A \cup A'$.

b) State and prove pasting lemma.

c) Show that the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = 2x + 3$ is a homeomorphism or not.

[6+5+5]

Q4) a) Let d be metric on X. Suppose $y \in B_d(x, \in)$ then show that there exists a $\delta > 0$ such that δ - ball centered at y is contained in $B_d(x, \in)$.

b) State and prove sequence lemma.

c) Let X_n be a metric space with metric d_n, for $n \in \mathbb{Z}_+$. Show that $\rho(x, y) = \max\{d_1(x_1, y_1), ..., d_n(x_n, y_n)\}$ is a metric for the product space $X_1 \times ... \times X_n$.

[6+6+4]

Q5) a) If $p : X \to Y$ is a quotient map, Z is a space and $p : X \to Z$ is a continuous map that is constant in each set $p^{-1}(y)$, for $y \in Y$, then prove that, g induces a continuous map $\delta : Y \to Z$ such that $f \circ g = g$.

b) Show that the usual topological space $(\mathbb{R}, U)$ is connected.

c) Show that the unit sphere $S^{n-1} = \{x \mid \|x\| = 1\}\, x$ in $\mathbb{R}^n$ is path connected.

[4+9+3]

Q6) a) Show that the relation "$x \sim y$" is an equivalence relation on X.

b) Prove that a topological space X is locally path connected if and only if for every open set U of X, each component of U is open in X.

c) State and prove Bolzano Weirstrass property.

[5+5+6]

Q7) a) State and prove Heine Borel theorem.

b) If (X,τ_1) and (Y,τ_2) be topological spaces then prove that the product space $(X\times Y;\tau)$ is compact if and only if (X,τ_1) and (Y,τ_2) are compact.

[8+8]

Q8) a) State and prove Lebesgue number lemma.

b) Let X be a locally compact Hausdorff space which is not compact and Y be the one-point compactification of X then prove that

i) Y is a compact Hausdorff space

ii) X is a subspace of Y

iii) The set Y - X consists of a single point and

iv) $\overline{X} = Y$

[8+8]

