MMDSC-3.2

Sl. No.

III Semester M.Sc. (CBCS) Degree Examination, October/November - 2025
MATHEMATICS
Topology - I

Time : 3 Hours **Maximum Marks : 80**

Answer any Five full questions.

1. a) Define the following:
i) Finite and infinite sets
ii) Countable and uncountable sets
iii) Discrete and indiscrete topological space

b) Let β be a basis for a topology on X. Let τ be defined by $\tau = \{U \subset X \mid$ for each $x \in U$ there is an element $B \in \beta$ such that $x \in B \subset U\}$ then prove that τ is a topology on X.

c) Show that the collection $\beta_1 = \{[a,b] \mid a < b, a,b \in \mathbb{R}\}$ the set of all closed open intervals in the real line $\mathbb{R}$. **(4+8+4)**

2. a) Define product topology on X×Y. If β is a basis for the topology of X and ζ is a basis for the topology of Y then show that the collision.
$D = \{B \times C \mid B \in \beta \text{ and } c \in \zeta\}$ is a basis for the topology of X×Y

b) Define order topology. Let X be an ordered set in the order topology, let Y be a subset of X that is convex in X. Then prove that the order topology on Y is the same as the topology Y inherits as a subspace of X. **(8+8)**

3. a) Prove that a subspace of a Hausdorff space is Hausdorff.

b) Let X and Y be topological spaces. Let $f : X \to Y$ then prove the following are equivalent
i) f is Continuous
ii) $f(\bar{A}) \subset \overline{f(A)}$.
iii) For every closed set B in Y, the set $f^{-1}(B)$ is closed in X.

c) If X_α be an indexed family of spaces. Let $A_\alpha \subset X_\alpha$ for each α. If $\prod_\alpha X_\alpha$ is given either the product or the box topology then prove that $\underset{\alpha \in J}{\pi} \bar{A}_\alpha = \overline{\underset{\alpha \in J}{\pi} A_\alpha}$. **(4+8+4)**

[P.T.O.

4. a) Let J(a,b) = min $\{|a-b|,1\}$ be the Standard bounded metric on $\mathbb{R}$.

If x and y are any two points of $\mathbb{R}^w$, *define* $D(x,y) = \text{lub}\left\{\frac{\bar{d}(x_i,y_i)}{i}\right\}$ then prove that D is a metric that induces the product topology on $\mathbb{R}^w$.

b) State and prove uniform limit theorem. **(8+8)**

5. a) Show that the closure of a connected set is connected.

b) If L is a linear combination in the ordered topology then prove that L is connected and so is every interval and ray in L.

c) Let Y_i be a collection of path-connected subspaces of X and $\cap Y_i \neq \phi$ then show that $Y = \cup Y_i$. **(5+7+4)**

6. a) Show that a component of X is connected and disjoint subsets of X whose union is X such that each connected subset of X intersects only one of them.

b) Define locally connected. Show that a topological space X is locally connected if and only if for every open set U *of* X each component of U is open in X.

c) Show that every component of a locally connected space is open. **(6+6+4)**

7. a) State and prove tube Lemma.

b) Show that Every closed and bounded subspace of $\mathbb{R}^n$ is compact. **(8+8)**

8. a) Let X be a metrizable space then prove the following are equivalent

i) X is compact

ii) X is limit point compact

iii) X is sequentially compact

b) Let X be locally compact Hausdorff and Y be a subspace of X. If Y is closed in X or open in X then prove that Y is locally compact. **(10+6)**
