MMDSC3.3

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III Semester M.Sc. Degree Examination, February - 2024
(SLM Scheme CBCS)
MATHEMATICS
Course - III : Functional analysis

Time : 3 Hours **Max. Marks : 80**

Instruction : ***Answer any five full questions.***

Q1) a) Let (x,d) be a complete metric space and (y,d) be a subspace of (x,d) then prove that y is complete if and only if y is closed.

b) State and prove Banach contraction mapping theorem.

[8+8]

Q2) a) State and prove cantor's intersection theorem.

b) Define complete and totally bounded. Let (x,d) be a complete and totally bounded metric space then prove that X is complete.

[8+8]

Q3) a) Define linear spaces. Let a linear space X be the sum of two subspaces M and N, so that X = M+N than prove that $X = M \oplus N \Leftrightarrow M \cap N = \{0\}$.

b) Define normed spaces. Prove that a normed linear space X is complete if and only if every absolutely convergent series in X is convergent.

[8+8]

Q4) a) State and prove Hahn - Banach theorem for any complex linear space.

b) If X is a compact Hausderff space then prove that $\subseteq (X.\mathbb{R})$ separates points of X.

[12+4]

Q5) a) State and prove stone weierstrass theorem for complex case.

b) Show that l_∞ is a Banach space.

[12+4]

Q6) a) State and prove closed graph theorem.

b) If T* is also a continuous linear transformation from N into F then prove the following.

i) $\|T^*\| = \|T\|$

ii) $(T_1 T_2)^* = T_2^* T_1^*$

iii) $I^* = I$

[12+4]

Q7) a) In a Hilbert space the parallelogram law holds than P.T

$$\| x + y \|^2 + \| x - y \|^2 = 2\left(\| x \|^2 + \| y \|^2\right)$$

b) State and prove Schwarz's inequality.

c) Define Hilbert space give an example.

[8+4+4]

Q8) a) State and prove Bessel's inequality.

b) State and prove Riesz Lemma.

[12+4]

