**MMDSC-3.3**

Sl.No. :

**Total No. of Pages : 2**

# III Semester M.Sc. Degree Examination, June/July - 2024
## (CBCS, SLM Scheme)
## MATHEMATICS
## Course - III : Functional Analysis

**Time : 3 Hours** **Max. Marks : 80**

***Instruction :*** ***Answer any five full questions :***

***Q1)*** a) Define an open ball and open set in a metric space. Show that a set G is open in a metric space (X, d) if and only if G is a union of open spheres.

b) State and prove Banach fixed point theorem.

**[8+8]**

***Q2)*** a) Show that every nonempty complete metric space is of second category.

b) State and prove Ascoli-Arzela theorem.

**[6+10]**

***Q3)*** a) Let a linear space X be the sum of two subspaces M and N, so that X = M + N then show that $X = M \oplus N \Leftrightarrow M \cap N = \{0\}$.

b) Define bounded linear operator. Show that the linear space $\beta(N, N')$ over F is a normed linear space with respect to $||T||=\text{Sup}\{||T(x)||:||x|| \leq |\}$.

**[8+8]**

***Q4)*** a) State and prove Hahn-Sanach theorem for normed linear space.

b) State and Stone-Weierstrass theorem for complex case.

**[10+6]**

***Q5)*** a) If $p > 1$, $q > 1$, $\frac{1}{p}+\frac{1}{q}=1$ and $a, b > 0$ then P.T $ab < \frac{a^p}{p}+\frac{b^q}{q}$.

b) State and prove open mapping theorem.

**[4+12]**

***Q6)*** a) Let X and Y be two Banach spaces and let Ti X $\rightarrow$ Y is a linear transformation form X into Y then P.T T is continuous if and only if $\Gamma(T)$ is closed.

b) If T* is also a catinuous linear transformation from N into F then P.T

i) $||T^*|| = ||T||$

ii) $(T_1\ T_2)^* = T_2^*\ T_1^*$.

**[12+4]**

***Q7)*** a) In a Hilbert space H, the innerproduct is continuous i.e, if $x_n \rightarrow x,\ y_n \rightarrow y$ as $n \rightarrow \infty$ then prove that $(x_n, y_n) \rightarrow (x, y)$ as $n \rightarrow \infty$.

b) Show that $l_2 = \{x = (x_1, x_2, \ldots\ldots, x_n, \ldots\ldots) : \sum_{n=1}^{\infty} |x_n|^2)^{1/2} < \infty, x_i \in f\}$ is a Hilbert Space.

**[8+8]**

***Q8)*** a) Let $\{e_i\}$ be an orthonermal set in Hilbert space H then prove the following are equivalent.

i) $\{e_i\}$ is complete

ii) $x \perp (e_i\} \Rightarrow x = 0$

iii) $x \in H \Rightarrow x = \Sigma(x, e_i)\, e_i$

iv) $x \in H \Rightarrow ||x||^2 = \Sigma|(x, e_i)|^2$

b) State Riesz representation theorem.

**[14+2]**

