MMDSC-3.3

Sl.No. :

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III Semester M.Sc. Degree Examination, May/June - 2026
(SLM Scheme, CBCS)
MATHEMATICS
Course - III : Functional Analysis

Time : 3 Hours **Max. Marks : 80**

Instruction : ***Answer any five full questions.***

Q1) a) Define a closed set in a metric space. Show that a set F is a closed set in a metric space (X, d) if and only if the complement of F is open.

b) State and prove picard's theorem.

[8+8]

Q2) a) Show that by using cantor's intersection theorem to prove Baire's Category theorem.

b) Define sequentially compact and slow that every sequentially compact metric space is both complete and totally bounded.

[8+8]

Q3) a) Let T : N $\rightarrow$ N′ be a linear operator from a normed linear space N into a nermed linear space N′ then prove the following are equivalent.

i) T is a continuous linear operator

ii) T is continuous at $x = 0$

iii) T is a bounded linear operator

iv) If $S = \{x \in N : \|x\| \leq 1\}$ is the closed unit sphere in N, then T(S) is bounded in N′.

b) Define normed spaces and bounded linear operator prove that a normed linear space X is complete if and only if every absolutely convergent series in X is convergent.

[8+8]

Q4) State and prove Hahn - Banach theorem for any real linear space. **[16]**

Q5) a) State and prove Holder's inequality.

b) If $T : X \to Y$ is a one to one, continuous linear transformation from X onto Y, then show that T is a homeomorphism from a Banach space X onto Banach space Y.

[8+8]

Q6) a) State and prove uniform bounded principle.

b) Show that if $T : X \to Y$ is a closed linear operator from a normed space X into a normed linear space Y then P.T. T(T) is a closed subspace of $X \times Y$.

[12+4]

Q7) a) If (H, (,)) is an inner product space, then prove that H is a normed linear space where $\|x\| = \sqrt{(x,x)}$, for all $x \in H$.

b) P.T. in a Helbert space H, $4(x,y) = \|x+y\|^2 - \|x-y\|^2 + i\|x+iy\|^2 - i\|x-iy\|^2$ where $i = \sqrt{-1}$.

[8+8]

Q8) a) Let M be a closed linear subspace of a Hilbert space H then P.T. $H = M \oplus M^{\perp}$.

b) State and prove Riesz Lemma.

[10+6]

