MMDSC-3.3

Sl. No.

III Semester M.Sc. Degree Examination, October/November - 2025

MATHEMATICS

Functional Analysis - I

(CBCS SLM Scheme)

Time : 3 Hours **Maximum Marks : 80**

Answer any Five full following questions.

1. a) Let *(x,d)* be a complete metric space and *(y,d)* be a subspace of *(x,d)* then prove that y is complete If and only if *y* is closed.

b) State and prove Picard's theorem. **(8+8)**

2. a) Define sequentially compact and show that every sequentially compact metric space is both complete and totally bounded.

b) State and prove Cantor's intersection theorem. **(8+8)**

3. a) Define linear spaces. Let a linear space X be the sum of two subspaces M and N, so that X=M+N then prove that $X = M \oplus N \Leftrightarrow M \cap N = \{0\}$.

b) Define normed spaces and bounded linear operator prove that a normed linear space X is complete if and only if every absolutely convergent series in X is convergent.**(8+8)**

4. a) State and prove Hahn-Banach theorem for any complex linear space.

b) State and prove Stone-Weierstrass theorem for Complex case **(8+8)**

5. a) Show that l_∞ is a Banach space

b) If $T : X \to Y$ is a one to one, continuous linear transformation from X on to Y, then show that T is a homomorphism from a Banach space X onto Banach space Y. **(8+8)**

[P.T.O.

6. a) State and prove uniform bounded principle.

b) Let N be a normed linear space ,then show that non empty subset X of N is bounded if and only if f(x) is a bounded set of numbers for every $f \in N^*$, where N* is the conjugate of N. **(8+8)**

7. a) In a Hilbert space the parallelogram law holds then prove that $\|x+y\|^2 + \|x-y\|^2 = 2\left(\| x \|^2 + \| y \|^2\right)$.

b) State and prove Riesz's Lemma. **(8+8)**

8. a) Let M be a closed linear subspace of a Hilbert space H then Prove that $H = M \oplus M^{\perp}$.

b) Let M be a closed proper subspace of a normed linear space N and $\varepsilon > o$, then show that there is a point x_{ε} on the unit sphere such that in f $\{\| x_0 - x_{\varepsilon} \| : x_0 \in M\} > 1 - \varepsilon$. **(8+8)**
