MMDSC-4.1

Sl.No. :

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Fourth Semester M.Sc. Degree Examination, May - 2026
(SLM Scheme) (CBCS)
MATHEMATICS
Measure and Integration

Time : 3 Hours **Max. Marks : 80**

Answer any five of the following questions.

Q1) a) Let $\{A_n\}$ be a countable collection of sets of real numbers then prove that

$$m^*\left(\bigcup_{n=1}^{\infty} A_n\right) \leq \sum_{n=1}^{\infty} m^*(A_n).$$

b) If A is countable set then show that m* (A) = 0.

c) If E_1 and E_2 are measurable sets then show that $E_1 \cup E_2$ is also measurable.

[6 + 5 + 5]

Q2) a) Let E be a measurable set of finite measure and $\{f_n\}$ a sequence of measurable functions defined on E. Let f be measurable real valued function such that for each $x \in E$, $f_n(x) \to f(x)$. Then show that for given $\varepsilon > 0$ and $\delta > 0$ there is a set $A \subset E$ with $m(A) < \delta$ and an integer N such that for all $x \notin A$ and for all $n \geq N$, $|f_n(x) - f(x)| < \varepsilon$.

b) Let $E \subset [0,1)$ be a Lebesgue measurable set then show that E + y is measurable and m (E + y) = m(E), Where $y \in [0,1)$.

[10 + 6]

Q3) a) If f and g are bounded measurable functions defined on a set E of finite measure, then show that

i) $\int_E af = a\int_E f$

ii) $\int_E (f+g) = \int_E f + \int_E g$

iii) If f = gac. then $\int_E f = \int_E g$

iv) $\int_{A\cup B} f = \int_A f + \int_B f$, where A and B are disjoint sets.

b) If f is measurable function and f = g a.c then show that g is measurable.

[10 + 6]

Q4) a) If $f\{f_n\}$ is a sequence of non-negative measurable functions and $f_n(x) \to f(x)$ ae. On a set E, then show that $\int f = \lim_{n\to\infty} \int f_n$.

b) Let f be non - negative function which is integrable over a measurable set E. Then show that for given $\varepsilon > 0$ there is $\delta > 0$ such that for every set $A \subset E$ with $m(A) < \delta$, $\int_A f < \varepsilon$.

[8 + 8]

Q5) a) Prove that the sum. difference and product of two functions of bounded variations over [a,b] is of bounded variation over [a,b].

b) Let f be an increasing and real valued function on the interval [a,b]. Then show that f is differentiable atmost Every where and $\int_a^b f'(x)dx = f(b) - f(a)$.

[8 + 8]

Q6) a) Let f be an integrable function on [a.b] and suppose that $F(x) = \int_a^x f(t)dt + F(a)$. Then show that F'(x) = $f(x)$ ae. on [a,b].

b) Prove that a function F is an indefinite integral if and only if it is absolutely continuous.

c) If f is absolutely continuous on $[a,b]$ and $f'(x)=0$ a.e. Then show that f is constant.

[6 + 5 + 5]

Q7) a) If $\{A_i\}$ is a sequence of sets and $A \subset \bigcup_{i=1}^{\infty} A_i$, then show that $\mu(A) \le \sum_{i=1}^{\infty} (A_i)$.

b) Let $\{E_i\}$ be a disjoint sequence of measurable sets and Let A be any set. Then show that $\mu^*(A \cap E) = \sum_{i=1}^{\infty} (A \cap E_i)$ where $E = \bigcup_{i=1}^{\infty} E_i$ **[8 + 8]**

Q8) a) Let E be a set in R σ δ with $(\mu \times \nu)(E) < \infty$ and $g(x) = \nu(E_x)$ then show that $\int g\, d\mu (\mu \times \nu)(E)$.

b) Let E be a set for which $(\mu \times \nu)(E) = 0$. Then show that for all x. $\nu(E_x) = 0$.

[8 + 8]

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