MMDSC-4.1

Sl.No. :

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Fourth Semester M.Sc. Degree Examination, November - 2024
(CBCS) (SLM Scheme)
MATHEMATICS
Measure and Integration

Time : 3 Hours **Max. Marks : 80**

Instructions : ***1) Attempt any Five full Questions.***
2) All questions carry equal marks.

Q1) a) Prove that for any collection C of subsets of X there is a smallest algebra A which contains C.

b) Prove that the Lebesgue measure of an interval is its length.

[8+8=16]

Q2) a) If $\{E_i\}$ is a sequence of Lebesgue measurable sets then show that

$$m\left(\bigcup_{i=1}^{\infty} E_i\right) \leq \sum_{i=1}^{\infty} m(E_i)$$

b) Let f and g be measurable real valued functions defined on same domain. Then show that following are measurable functions (where c is constant).

i) $f + c$

ii) $f + g$

iii) $f - g$

iv) f^2.

[8+8=16]

Q3) a) If f and g are bounded measurable functions defined on a set E of' finite measure then, show that

i) $\int_E af = a \int_E f$

ii) $\int_E (f + g) = \int_E f + \int_E g$

iii) if $f = g$ ae. then $\int_E f = \int_E g$

iv) if $f \leq g$ ae. then $\int_E f \leq \int_E g$

b) Let $\{f_n\}$ be a sequence of measurable functions defined on a set E of finite measure. Suppose $|f_n(x)| \leq M$ for all n and for all $x \in E$. If $\lim_{n\to\infty} f_n(x) = f$ for each $x \in E$, then Show that $\lim_{n\to\infty} \int_E f_n = \int_E f$

[8+8=16]

Q4) a) If $\{f_n\}$ is a sequence of non-negative measurable functions and $f_n(x) \to f(x)$ ae. on a set E, then show that $\int f = \lim_{n\to\infty} \int f_n$.

b) If f is measurable function then show that its positive part f^+ and negative part f^- are also measurable.

[8+8=16]

Q5) a) Let f be an increasing and real valued function on the interval $[a, b]$. Then show that f is differentiable almost everywhere and $\int_a^b f'(x)\,dx = f(b) - f(a)$.

b) Prove that a function f is of bounded variation over $[a, b]$ if and only if f is the difference of two monotone real valued functions on $[a, b]$.

[8+8=16]

Q6) a) If f is integrable On $[a, b]$ and $\int_a^x f(t)\,dt = 0$ for all $x \in [a, b]$ then show that $f(t) = 0$ ae. on $[a,b]$.

b) Prove that an absolutely continuous function is of bounded variation.

c) If f is absolutely continuous on $[a, b]$ and $f'(x) = 0$ ae. then show that f is constant.

[6+5+5=16]

Q7) a) Let $\{E_i\}$ be a disjoint sequence of measurable sets and let A be any set. Then show that $\mu^*(A \cap E) = \sum_{i=1}^{\infty} (A \cap E_i)$ where $E = \bigcup_{i=1}^{\infty} E_i$

b) If $\{A_i\}$ is a sequence of sets and $A \subset \bigcup_{i=1}^{\infty} A_i$, then show that $\mu(A) \leq \sum_{i=1}^{\infty} (A_i)$.

[8+8=16]

Q8) a) Let $\{(A_i \times B_i\}$ be a countable disjoint collection of measurable rectangles whose union is a measurable rectangle $A \times B$. Then show that $\lambda(A \times B) = \Sigma\lambda(A_i \times B_i)$.

b) Let μ and v be σ-finite outer measures on X and Y respectively. For any non-negative $(\lambda \times v)$ - measurable function show that

$$\int_{X \times Y} f(x, y) d(\mu \times v)(x, y) = \int_Y \left(\int_X f(x, y) d_\mu(x) \right) d_v(y)$$

[8+8=16]