MMDSC-4.1

Sl.No. :

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Fourth Semester M.Sc. Degree Examination, October - 2025
(SLM Scheme) (CBCS)
MATHEMATICS
Measure and Integration

Time : 3 Hours **Max. Marks : 80**

Answer any five of the following questions.

Q1) a) Prove that the Lebesgue measure of an interval is its length.

b) Prove that

i) If E is measurable set, then its complement E^c is also measurable set.

ii) Empty set is measurable.

[8 + 8]

Q2) a) Let $\{E_n\}$ be an infinite decreasing sequence of Lebesgue measurable sets, that is $E_{i+1} \subset E_i$, i =1,2,..... Let $m(E_i)$ be finite then prove that $m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} m(E_n)$.

b) Let f and g be measurable real valued functions desined on same domain. Then show that following are measurable functions (where C is constant).

i) $f + C$ ii) Cf

iii) $f + g$ iv) f^2

[8 + 8]

Q3) a) Let $\{f_n\}$ be a sequence of measurable functions defined on a set E of finite measure. Suppose $|f_n(x)| \leq M$ for all n and for all $x \in E$. If $\lim_{n\to\infty} f_n(x) = f$ for each $x \in E$, then show that $\lim_{n\to\infty} \int_E f_n = \int_E f$.

b) Define characteristic function of a set. Let ϕ and ψ be the simple functions which vanish outside the set of finite measure then show that $\int (a\,\phi + b\,\psi) = a\int \phi + b\int \psi$

[8 + 8]

Q4) a) If f is measurable function then show that its positive part f^+ and negative part f^- are also measurable.

b) Let $\{f_n\}$ be a increasing sequence of non- negative measurable functions and Let $\lim_{n\to\infty} f_n(x) = f(x)$ then show that $\int f = \lim_{n\to\infty} \int f_n$.

[8 + 8]

Q5) a) Let f be an increasing and real valued function on the interval [a,b]. Then show that f is differentiable almost every where and $\int_a^b f'(x)dx = f(b) - f(a)$.

b) Prove that a function f is of bounded variation over [a,b] if and only if f is the difference of two monotone real valued functions on [a,b].

[8 + 8]

Q6) a) If f is bounded and measurable on [a,b] and $F(x) = \int_a^b f(t)dt + F(a)$, then show that $F'(x) = f(x)$ a.e. in [a,b].

b) Prove that an absolutely continuous function is bounded variation.

c) If f is absolutely continuous on [a,b] and $f'(x) = 0$ a.e. then show that f is constant.

[6 + 5 + 5]

Q7) a) Let $\{E_i\}$ be a disjoint sequence of measurable sets and Let A be any set. Then show that $\mu^*(A \cap E) = \sum_{i=1}^{\infty} (A \cap E_i)$ where $E = \bigcup_{i=1}^{\infty} E_i$,

b) If $\{A_i\}$ is a sequence of sets and $A \subset \bigcup_{i=1}^{\infty} A_i$, then show that $\mu(A) \le \sum_{i=1}^{\infty} A_i$.

[8 + 8]

Q8) a) Let $\{(A_i \times B_i)\}$ be a countable disjoint collection of measurable rectangles whose union is a measurable rectangle $A \times B$. Then show that $\lambda(A \times B) = \sum \lambda(A_i \times B_i)$.

b) Let μ and ν be σ - finite outer measures on x and y respectively. For any non-negative $(\lambda \times \nu)$ - measurable function f, show that

$$\int_{x\times y} f(x,y)d(\mu\times\nu)(x,y) = \int_x \left(\int_y f(x,y)d_\nu(y) \right) d_\mu(x).$$

[8 + 8]

