MMDSC-4.2

Sl.No. :

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Fourth Semester M.Sc. Degree Examination, May - 2026
(SLM Scheme) (CBCS)
MATHEMATICS
Topology - II

Time : 3 Hours **Max. Marks : 80**

Instructions : 1) Answer any Five full questions.
2) All questions carry equal marks.

Q1) a) Define T_0 and T_1 spaces. Show that a topological space Cx, τ is a T_0 - space. If and only if the closures of distinct points are distinct.

b) Suppose that X has a countable basis. Then show that

i) Every open covering of X contains a countable subcollection covering X.

ii) There exists a countable subset of X that is dense in X.

[8 + 8]

Q2) a) Show that a topological space (X, τ) is regular if and only if given any $x \in X$ and open set G containing x, f open set G^* in X, $\exists: x \in G^* \subseteq \overline{G}^* \subseteq G$.

b) Show that the normal property is topological but not hereditary.

[8 + 8]

Q3) State and prove Urysohn lemma. **[16]**

Q4) a) Define m=manifold. If X is a compact m-manifold then prove that X can be imbedded in $\mathbb{R}^n$ for some positive integer N.

b) State

i) Tietze's extension theorem

ii) Partition of unity

[12 + 4]

Q5) State and prove the Tychonoff theorem. **[16]**

Q6) a) Let X be a metrizable Space. If A is an open covering of X, then show that there is a collection D of subset of X such that

i) D is an open covering of X

ii) D is a refinement of A

iii) D is countably locally finite

b) Show that every closed subspace of a paracompact space is paracompact.

[11 + 5]

Q7) a) Show that the relations $\simeq$ and $\simeq$p are equivalence relations.

b) If X is a topological space and $x_0 \in X$, then show that $\pi_1 (x, x_0)$ is a group with respect to the operation $*$.

[10 + 6]

Q8) a) Show that the map $P : R \to S'$ given by the equation $P(X) = (\cos 2\pi x, \sin 2\pi x)$ is a covering map.

b) Explain the fundamental group of the cycle is infinite cycle.

c) Define essential and inessential maps. Let $h : X \to Y$. If h is inessential, then show that h* is the zero homomorphism.

[6 + 5 + 5]

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