# MMDSC-4.2

Sl.No. :

**Total No. of Pages : 2**

## Fourth Semester M.Sc. Degree Examination, November - 2024
## (CBCS) (SLM Scheme)
## MATHEMATICS
## Topology - II

**Time : 3 Hours** **Max. Marks : 80**

**Answer any five full questions :**

***Q1)*** a) Define first and second countability axiom. Let X be a space satisfying the first countability axiom then show that the function $f : x \to y$ is continuous if and only if for every convergent sequence $\{x_n\}$ in X, converging to $x$ the sequence $\{f(x_n)\}$ converges to $f(x)$.

b) Define $T_0$ and $T_1$ spaces. Show that a topological space $(X, \tau)$ is a $T_0$ - space if and only if the closures of distinet points are distinct.

**[8+8]**

***Q2)*** a) Show that every compact subset of a $T_2$ - space is closed. **[8]**

b) Show that the normal property is topological but not hereditary. **[8]**

***Q3)*** State and prove Urysohn Lemma. **[16]**

***Q4)*** a) Define m-manifold. If X is a compact m-manifold, then prove that X can be imbedded in $\mathbb{R}^n$ for some positive integer N. **[12]**

b) State **[4]**

(i) Urysohn's Metrization Theorem.

(ii) Partition of unity

***Q5)*** a) Define the maximum principle.

b) Define completely regular space. Show that a product of completely regular spaces is completely regular.

**[4+12]**

***Q6)*** a) Let X be a metrizable space. If $\mathcal{A}$ is an open covering of X, then show that there is a collection $\mathcal{D}$ of subsets of X such that

i) $\mathcal{D}$ is an open Couring of X

ii) $\mathcal{D}$ is a refinement of $\mathcal{A}$

iii) $\mathcal{D}$ is countably locally finite

b) Define (i) a locally finite set with an example

(ii) countably locally finite set with an example.

**[12+4]**

***Q7)*** a) Show that the relations $\simeq$ and $\simeq_p$ are equivalence relations.

b) Show that the map $p : \mathbb{R} \to S'$ given by the equation $p(x) = (\cos 2\pi x, \sin 2\pi x)$ is a covering map.

**[10+6]**

***Q8)*** a) State and prove fundamental theorem of Algebra.

b) Explain the fundamental group of the cycle is infinite cycle.

**[12+4]**