MMDSC-4.2

Sl.No. :

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Fourth Semester M.Sc. Degree Examination, October - 2025
(SLM Scheme) (CBCS)
MATHEMATICS
Topology - II

Time : 3 Hours **Max. Marks : 80**

Instruction : Answer any five questions.

Q1) a) Define first and second countability axiom. Let X be a space satisfying the first countability axiom then show that the function $f : X \to Y$ is contenuous if and only if for every convergent sequence $\{x_n\}$ in X, converging to X the sequence $\{f(x_n)\}$ converges to $f(x)$.

b) Show that the first axiom space (first countability axiom) is both hereditary and topological.

[8 + 8]

Q2) a) Show that every compact subset of a T_2 - space is closed.

b) Show that every regular lindelöf space is normal.

[8 + 8]

Q3) State and prove Urysohn Lemma. **[16]**

Q4) State and prove Urysohn metrization theorem. **[16]**

Q5) a) Define completely regular space. Show that the product of completely regular spaces is completely regular.

b) Let X be a space. Then show that the following are equivalent :

i) X is completely regular

ii) X is homeomorphic to a subspace of a compact Hausdorff space

iii) X is homeomorphic to a subspace of a normal space

[10 + 6]

Q6) a) Let X be regular. Then show that the following conditions on X are equivalent. Every open covering of X has a refinement that is :

i) An open covering of X and countably locally finite.

ii) A covering of X and locally finite.

iii) A closed covering of X and locally finite

iv) An open covering of X and locally finite.

b) Define

i) A locally finite set with an example.

ii) Countably locally finite set with an example.

[12 + 4]

Q7) a) The operation * is well defined on path homotopy classes. Then show that it has the following properties.

i) (Associativity) if $[f]^*$ ($[g]$ * $[h]$) is defined, then so is ($[f]$ * $[g]$) * $[h]$ and they are equal.

ii) (Right and left identities) Give $x \in X$, let e_x denote the constant path $e_x : I \times X$ carrying all of I to the point x. If f is a path in X from x_0 to x_1, then $[f] * [e_{x_1}] = [f]$ and $[e_{x_1}] * [f] = [f]$

iii) (Inverse). Given the path f in X from x_0 to x_1, let $\tilde{f}$ be the path defined by $\tilde{f}(S) = f(1 - S)$. It is called the reverse of f. Then

$[f] * [\tilde{f}] = [e_{x_0}]$ and $[\tilde{f}] * [f] = [e_{x_1}]$

b) If $h : (X, x_0) \to (Y, y_0)$ is a homeomorphism of X with Y, then show that h* is an isomorphism of $\pi_1 (X, x_0)$ with $\pi_1 (Y, y_0)$.

[12 + 4]

Q8) a) State and prove fundamental theorem of Algebra.

b) Let $h : X \to Y$. If h is inessential, then show that $h*$ is the zero homomorphism.

[12 + 4]

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