MMDSC-4.3

Sl.No. :

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Fourth Semester M.Sc. Degree Examination, May - 2026
(SLM Scheme) (CBCS)
MATHEMATICS
Lattice Theory

Time : 3 Hours **Max. Marks : 80**

Instructions : 1) Answer any Five full questions.
2) All questions carry equal marks.

Q1) a) Let $f : P_1 \to P_2$ be an order isomorphism of posets P_1 and P_2. Let $R_1 \subseteq P_1$ be such that inf R_1 = a exists in P_1. Let $R_2 = f(R_1)$ ie., $R_2 = \{f(x) : x \in R_1\} = \{y \in P_2 : y = f(x)$ for some $x \in R_1\}$. Then prove that inf R_2 = b exists in P_2 and b = f(a).

b) Show that the notions of minimal elements and minimum coincide for chains.

[8 + 8]

Q2) a) Let D_n denote the set of all positive divisors of $n \in N$. Then show that $< D_n, | >$ is a sub poset of $< N, | >$. Further prove that $< D_n, | >$ is self dual for any $n \in N$.

b) Define the length of a poset. Is every poset of finite length poset? What about the converse? Justify.

c) Show that any poset P is order isomorphic with a collection of P ordered by inclusion.

[6 + 5 + 5]

Q3) a) Show that a non-empty intersection of an ideal and a dual ideal in a lattice is a convex sublattice. Conversely prove that every convex sublattice can be uniquely expressed in this form.

b) Let $(P, \leq)$ be a poset in which every pair of elements have a glb. Define a binary operation $\wedge$ on P as follows. For a, b $\in$ P, define a $\wedge$ b = glb {a, b}. Show that $(P, \wedge)$ is a semilattice.

[8 + 8]

Q4) a) Prove that a one to one map f of a Lattice L_1 onto a lattice L_2 is an isomorphism if and only if f and f^{-1} are order preserving.

b) Show that every ideal of a Lattice is principal if and only if L satisfies acc.

c) Prove that the set union of members of any ideal chain of a Lattice is again an ideal of L.

[8 + 4 + 4]

Q5) a) Define a conditionally complete lattice. Prove that a Lattice L is conditionally complete If and only if every non-empty subset of L bounded above in L has a join.

b) Prove that every homomorphic image of a modular lattice is modular.

[10 + 6]

Q6) a) Prove that lattice L is distributive if and only if mod (x,y,z) exists for every triplet x,y,z of elements of L.

b) State and prove isomorphism theorem for modular lattices.

[8 + 8]

Q7) a) State and prove Dedekind's Modularity criteria.

b) Prove that Every complemented modular lattice is relatively complemented.

c) Prove that a lattice L is modular if and only if $I(L)$ is modular.

[8 + 4 + 4]

Q8) a) Show that a Boolean algebra B is complete and atomic if and only if B is isomorphic with the subset lattice of some set.

b) Prove that every complete Boolean algebra is infinitely distributive.

[10 + 6]

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