MMDSE-1.4

Sl.No. : **Total No. of Pages : 2**

I Semester M.Sc. Degree Examination, May/June - 2026
(SLM Scheme, CBCS)
MATHEMATICS
Course-IV : Ordinary Differential Equations

Time : 3 Hours **Max. Marks : 80**

Instructions : 1) Answer any five full questions.
2) All questions carry equal marks.

Q1) a) Differentiate between Homogeneous and Non-Homogenous Second order Linear ordinary differential equations.

b) If $y_1(x)$ and $y_2(x)$ are two solutions of $y''+P(x)y'+Q(x)y = 0$ on $[a,b]$. Then prove that $y_1(x)$ and $y_2(x)$ are Linearly independent iff $w(y_1, y_2) \neq 0$.

[6+10]

Q2) a) Show that $y = c_1x+c_2x^2$ is the general solution of $x^2y'' - 2xy'+2y = 0$.On any interval not containing zero and find the particular solution for which $y(1) = 3$ and $y'(1) = 5$.

b) Find the general solution of the differential equation $y''+y = 0$ given that $y_1 = \sin x$.

[10+6]

Q3) a) Find the Wronskian of the independent solutions of $y''' - y'' - y' + y = 0$.

b) Find the adjoint differential equation to $x^2y'' - 3xy' + 3y = 0$.

c) Explain briefly the method of variation of parameters.

[8+4+4]

Q4) a) State and prove Sturm's Separation theorem.

b) Express $x^3 + x^2 + x + 1$ in terms of Legendre's polynomial.

[12+4]

Q5) a) Prove that $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}}\cos x$.

b) Prove that $\int_0^\infty e^{-x} L_m(x) L_n(x) dx = \delta_{m,n} = \begin{cases} 1, & \text{when } m = n \\ 0, & \text{when } m \neq n \end{cases}$.

c) Express $3x^2 + 2x + 4$ in Laguerre polynomial.

[4+8+4]

Q6) a) Find the general solution for the homogenous linear system of equation

$$\frac{dx}{dt} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \text{X}.$$

b) Explain the method to find the nature of critical points, stability and paths.

[12+4]

Q7) a) State and prove Picard's theorem for existence and uniqueness.

b) State and prove orthogonal property of Eigen functions.

[8+8]

Q8) a) Solve by Green's function method

$$\frac{d^2y}{dx^2} + 4y = f(x), x \in [0,1], y(0) = y'(1) = 0.$$

b) Employ Picard's method to obtain the solution of the differential equation $\frac{dy}{dx} = x^2 + y^2$, given that $y = 0$ when $x = 0$. Hence find $y(0.1)$ correct to four decimal places.

[8+8]

