MMDSE-1.4

Sl.No. : **Total No. of Pages : 2**

I Semester M.Sc. Degree Examination, June-2025
(CBCS, SLM Scheme)
MATHEMATICS
Course - IV: Ordinary Differential Equations

Time : 3 Hours **Max. Marks : 80**

Instruction : Answer any five full questions.

Q1) a) Define ordinary differential equation and give two examples.

b) Find the particular solution of $y'' - y = 0$ given condition $y(0) = 1, y'(0) = 1$.

c) Show that e^x and e^{-x} are two Linearly independent solution of $y'' - y = 0$ on any interval.

[5+5+6]

Q2) a) If $y_1(x)$ and $y_2(x)$ are any two solutions of $y'' + P(x)y' + Q(x)y = 0$ on [a, b] then prove that their Wronskian $w(x) = w(y_1 y_2)$ is either identically zero or never zero.

b) Find the Wronskian of the independent solutions of $y''' - y'' - y' + y = 0$.

[10+6]

Q3) a) Find the adjoint differential equation to $x^2 y'' - 3xy' + 3y = 0$.

b) Solve by the method of variation of parameters $x^2 y'' - xy' - 3y = x^3$.

c) Use the method of reduction of order to solve the differential equation $y'' = y'.e^y$ with initial conditions $y(0) = 0$ and $y'(0) = 1$.

[5+5+6]

Q4) a) Prove that $T_{m+n}(x) + T_{m-n}(x) = 2\,T_m(x)\,T_n(x)$.

b) Prove that $P_n(x) = \frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$.

c) Express $x^3 + x^2 + x + 1$ in terms of Legendre's polynomial.

[4+8+4]

Q5) a) Find the general solution for the homogenous Linear system of

$$\frac{dx}{dt} = \begin{pmatrix} 1 & -1 & -1 \\ 1 & 3 & 1 \\ -3 & -6 & 6 \end{pmatrix} x.$$

b) Prove that $L'_n(x) = L'_{n-1}(x) - L_{n-1}(x)$.

[12+4]

Q6) a) Find all critical points and their nature and stability of the non-Linear system $\frac{dx}{dt} = 8x - y^2$ and $\frac{dy}{dt} = -6y + 6x^2$.

b) Define critical point, path, Isolated critical point and saddle point.

[12+4]

Q7) a) Solve by Greener's method for the differential equation $\frac{d^2y}{dx^2} + y = 1$ with boundary condition $y(0) + y'(0) = 0$, $y\left(\pi/2\right) + y'\left(\pi/2\right) = 0$.

b) Find the Eigen values and Eigen functions of $y'' + \lambda y = 0$, $y(0) = 0 = y(\pi)$ by using Sturm Liouville's problem.

[8+8]

Q8) a) State and prove orthogonality property of Eigen functions.

b) Use the Picard's method to obtain the fourth approximation to the solution of $\frac{dy}{dx} = x + y,\ y(0) = 1$ and hence find y at $x = 0.1, 0.2$.

[8+8]

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