MMDSE-1.4

Sl. No.

I Semester M.Sc. Degree Examination, October/November - 2025
MATHEMATICS
Ordinary Differential Equations
Course : IV
(CBCS, SLM Scheme)

Time : 3 Hours **Maximum Marks : 80**

***Instructions to Candidates*:**

Answer any Five full questions.

1. a) Define ordinary differential equation and give two examples.

b) If $y_1(x)\ and y_2(x)$ are two solution of $y''+P(x)y'+Q(x)y=0\ on\ [a,b]$. Then prove that $y_1(x)\ and\ y_2(x)$ are Linearly independent iff $w(y_1y_2)\neq 0$. **(4+12)**

2. a) Find the general solution of the differential equation $y_1=e^x,\ y_2=e^{-2x}\ of\ y''+y'-2y=0\ with\ y(1)=0,\ y'(1)=0$

b) Explain briefly the method of variation of parameters. **(8+8)**

3. a) Find the Wronskian of the independent solutions of $\left(D^3-D^2-D+1\right)y=0$.

b) Find the adjoint differential equation to $x^2y''-3xy'+3y=0$ **(8+8)**

4. a) Obtain the Legendre's Polynomial $P_n(x)$ using series solution of Legendre's differential equation.

b) Prove that $T_n^2(x)-T_{n+1}(x)T_{n-1}(x)=(1-x^2)$ **(12+4)**

[P.T.O.

5. a) Show that $J_{-n}(x) = (-1)^n J_n(x)$

b) Find the general solutions for the homogeneous linear system of $\frac{dx}{dt} = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & -4 \\ 4 & 1 & -4 \end{pmatrix} x$.

(4+12)

6. a) Find all the critical points with nature and stability of the non linear system:

$$\frac{dx}{dt} = 1 - xy, \frac{dy}{dt} = x - y^3$$

b) Define stability of critical point, Focal point, Node and centre **(12+4)**

7. a) State and prove Sturm's comparision theorem.

b) Find the Eigen value and Eigen functions of $y'' + \lambda y = 0$; $y(0) = 0, y(\pi) = 0$ **(8+8)**

8. a) Use the Picard's method to obtain the solution of the differential equation.

$\frac{dy}{dx} = x^2 + y^2$, given that y = 0 , when x = 0. Hence find y(0.1) correct to four decimal places.

b) Obtain Eigen value and Eigen functions of Laguerre equation. **(8+8)**
