MMDSE-2.4

Sl. No.

II Semester M.Sc. Degree Examination, May/June - 2025
MATHEMATICS
Partial Differential Equation
(CBCS SLM Scheme)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

Answer any **Five** full questions.

1. (a) Find the partial differential equation by eliminating the arbitrary constants $z = (x-a)^2 + (y-b)^2$.

(b) Solve $p\sqrt{x} + q\sqrt{y} = \sqrt{z}$.

(c) Find the orthogonal trajectories of $y^2 = cx^3$ where c is an arbitrary constant. **(5+5+6)**

2. (a) Find the Cauchy problem for $uu_x + uu_y = 0$ $u(x, 0) = 0, 0 \leq x \leq 1$.

(b) Solve the equation $\left(D^2 + 3DD' + 2D'^2\right)u = x + y.$

(c) Solve Pfaffian equation $xdy - ydx = 0$. **(6+5+5)**

3. (a) Solve $(p^2 + q^2)y = qz$ by Charpit's method.

(b) Find a complete integral of $yzp^2 - q = 0$.

(c) Solve $p^2x + q^2y = z$ by Jacobi's method. **(6+6+4)**

4. (a) Classify the equation $\frac{\partial^2 u}{\partial x^2} = x^2 \frac{\partial^2 u}{\partial y^2}$ and reduce it to the canonical form.

(b) Classify the partial differential equation $Z_{xx} + Z_{yy} = 0$. **(8+8)**

5. (a) Solve $\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial r^2} = 0$.

(b) Solve the Boundary value problem $\frac{\partial u}{\partial x} = 4\frac{\partial u}{\partial y}$ if $u(0, y) = 8e^{-3y} + 4e^{-3y}$. **(8+8)**

[P.T.O.

6. (a) Prove that $u(r, \theta) = \frac{1}{2\pi} \int_0^{2\pi} \frac{(a^2 - r^2) f(\theta)}{\left[a^2 - 2ar\cos(\phi - \theta) + r^2\right]} d\phi$.

(b) Prove that $\int_{-\infty}^{\infty} \delta'(t-a) f(t)\, dt = -f'(a)$. **(12+4)**

7. (a) Solve $u_{tt} = c^2 u_{xx}$, $-\infty < x < \infty$, $0 < t < \infty$.

(b) If the Dirichlet problem for a bounded region has a solution then prove that it is unique. **(6+10)**

8. (a) Solve by Greens function method $y'' + 4y = f(x)$, $\forall x \in \left[0, \pi/2\right]$, $y(0) = y'\left(\pi/2\right) = 0$.

(b) Solve $y'' + \lambda y = 0$, $0 < x < L$ with $y(-L) = y(L)$ and $y'(-L) = y'(L)$ by using Periodic Eigen value problem. **(8+8)**
