# MMDSE-2.5

Sl.No. :

**Total No. of Pages : 3**

## Second Semester M.Sc. Degree Examination, May - 2026
## (SLM Scheme) (CBCS)
## MATHEMATICS
## Discrete Mathematics

**Time : 3 Hours** **Max. Marks : 80**

**Instructions : 1) Answer any Five full questions.**

**2) All questions carry equal marks.**

***Q1)*** a) Define tautology, contradiction and contingency statements. Identify whether the given statements are tautology, contradiction of contingency.

i) $p \wedge (\sim p \wedge q)$

ii) $[(p \wedge q) \vee \sim r] \leftrightarrow p$

iii) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

b) Define Principle Disjunctive Normal Form (PDNF), Principle Conjunctive Normal Form (PCNF).

i) Obtain PDNF of $[q \vee (p \vee \sim q)]$

ii) Obtain PCNF of $(\sim p \rightarrow r) \wedge (q \leftrightarrow p)$

**[8 + 8]**

***Q2)*** a) Test whether the following argument is valid $(p \rightarrow q) \wedge (r \rightarrow s) \wedge (p \vee r) / \therefore (q \vee s)$

b) Define negation of quantified statement. Then negate and simplify the following quantified statement

i) $\exists x, [p(x) \rightarrow \sim q(x)]$ ii) $\forall x, [\sim p(x) \wedge \sim q(x)]$

c) Establish the following results by using the principles of mathematical induction.

i) $1^2 + 2^2 + 3^2 + ..... + n^2 = \dfrac{n(n+1)(2n+1)}{6}$

ii) $1^3 + 2^3 + 3^3 + ..... + n^3 = \dfrac{n^2(n+1)^2}{4}$

**[5 + 5 + 6]**

***Q3)*** a) Explain the principle of inclusion-exclusion.

b) State and prove Pigeonhole principle. Show that in a group of 13 children, there must be atleast two children who were born in the same month.

c) Explain restricted permutation. Prove that the number of ways in which $n$ books can be arranged on a shelf so that two particular books are never together is $(n-2) \times (n-1)$ !

**[5 + 5 + 6]**

***Q4)*** a) Prove that ${}^{n-1}C_r = {}^{n}C_r + {}^{n}C_{r-1}$, $(0 \le r \le n)$.

b) Solve the following recurrence relation $y_{n+2} - y_{n+1} - 2y_n = n^2$.

c) Solve the recurrence relation $a_{n-2} - 5a_{n-1} + 6a_n = 2$ with initial condition $a_0 = 1$ and $a_1 = -1$.

**[4 + 6 + 6]**

***Q5)*** a) Determine the numeric function corresponding to the given generating functions.

i) $\dfrac{1}{5-6x+x^2}$ ii) $\dfrac{(1+x)^2}{(1-x)^4}$

b) State and prove Towers of Hanoi problem.

**[8 + 8]**

***Q6)*** a) Define equivalence relation. If R is the relation on Z, the set of integers such that R = {(x, y)/$x$ – y is divisible by 5}, show that R is an equivalence relation.

b) Let A = {1, 2, 3, 4}, B = {$p$, $q$, $r$, $s$} and C = {$x$, $y$, $z$} and also R = {(1, $p$), (1, $r$), (1, $s$), (2, $r$), (2, $s$), (3, $p$), (4, $s$)} and S = {($p$, $x$), ($p$, $z$), ($q$, $y$), ($r$, $y$), ($r$, $z$), ($s$, $x$), ($s$, $z$)}. Find the composition of the following

i) R ∘ S ii) R ∘ R

iii) S ∘ R iv) R ∘ R ∘ R

v) R ∘ R ∘ S vi) S ∘ S ∘ S

**[8 + 8]**

***Q7)*** a) If $R^{-1}$ and $S^{-1}$ are the inverses of the relations R and S respectively, then prove that $(RoS)^{-1} = S^{-1}$ o $R^{-1}$.

b) Define totally ordered set. Show that the relation '<' (less than) defined on N, the set of positive integers is neither an equivalence relation nor partially ordered relation but is a totally ordered relation.

c) Define dual of a POSET and prove that the dual of a poset is also a poset.

**[5 + 5 + 6]**

***Q8)*** a) Define minimal and maximal element of a POSET. Find the minimal and maximal elements of the POSET (P, ≤), where P = {1, 2, 3, 4, 6, 8, 12} and ≤ is the relation of divisibility.

b) State the properties of bounded lattices. Prove that, every finite lattice L=$\{a_1, a_2, \ldots a_n\}$ is bounded.

c) Prove the following :

i) Every chain is a distributive lattice

ii) In a distributive lattice, if an element has a complement then this complement is unique.

**[4 +4 + 8]**

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