MMDSE-2.5

Sl. No.

II Semester M.Sc. Degree Examination, May/June - 2025
MATHEMATICS
Discrete Mathematics
(CBCS SLM Scheme)

Time : 3 Hours **Maximum Marks : 80**

Instructions to Candidates:

1. Answer any **Five** full questions.
2. All questions carry **equal** marks.

1. a) Define logical connections. Construct the truth tables for the following statements.

i) Distributive laws.

ii) De' Morgans laws.

b) Without using truth table prove the following propositions are equivalent.

i) $(\neg p \vee q) \wedge (p \wedge (p \wedge q)) \equiv p \wedge q$.

ii) $\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$. **(8+8)**

(OR)

2. a) Test whether the following arguments are valid.

i) $p \rightarrow r$, $q \rightarrow r$ / $\therefore (p \vee q) \rightarrow r$

ii) $p \rightarrow q$, $r \rightarrow s$, $p \vee r$ / $\therefore q \vee s$

iii) $(\neg p \vee \neg q) \rightarrow (r \wedge s)$, $r \rightarrow t$, $\neg t$ / $\therefore p$

b) Define Principal Disjunctive Normal Form (PDNF) and Principal Conjunctive Normal Form (PCNF)

i) Obtain PDNF of $p \rightarrow \left[(p \rightarrow q) \wedge \neg(\neg q \vee \neg p)\right]$.

ii) Obtain PCNF of $(\neg p \rightarrow \neg r) \wedge (q \leftrightarrow p)$. **(8+8)**

[P.T.O.

3. a) By mathematical induction prove that for all positive integers $n \geq 1$,

$$1^2 + 3^2 + 5^2 + \ldots\ldots + (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1).$$

b) State and prove Pigeon hole principle and Extended Pigeon hole principle.

c) Prove that every integer $n \geq 2$ is prime or a product of primes. **(5+8+3)**

4. a) How many different numbers of 3 digits can be formed from the digits 1, 2, 3, 4, 5 and 6 if digits are not repeated? What will happen if repetitions are allowed?

b) Solve the recurrence relation of the Fibonacci sequence of numbers $f_n = f_{n-1} + f_{n-2}$, $n \geq 2$ with the initial condition $f_0 = 1, f_1 = 1$.

c) Solve the recurrence relation $y_{n+2} - y_{n+1} - 2y_n = n^2$. **(4+6+6)**

5. a) Determine the numeric function corresponding to the following generating functions:

i) $\dfrac{(1+x)^2}{(1-x)^4}$ ii) $\dfrac{x^4}{1-2x}$

b) State and prove Towers of Hanoi problem. **(8+8)**

6. a) Define the equivalence relation. Let R = {(*x*, *y*) : *x*-*y* is divisible by 3}. Then show that R is an equivalence relation on the set of integers Z.

b) i. Let A, B, C and D be four non-empty sets. Let R, S and T be the three relations from A to B, B to C and C to D respectively, Then prove that $(R \circ S) \circ T = R \circ (S \circ T)$.

ii. If R^{-1} and S^{-1} are the inverse of the relation R and S respectively, then prove that $(R \circ S)^{-1} = S^{-1} \circ R^{-1}$.

c) Define POSET, totally ordered set, well ordered set and lexicographic ordering, write suitable example for each of these sets. **(5+6+5)**

7. a) Find the minimal and maximal elements of the poset $(P, \leq)$, where P = {1, 2, 3, 4, 6, 8, 12} and $\leq$ is the relation of divisibility.

b) Prove that a poset has atmost one greatest element and atmost one least element.

c) Define dual of a POSET and prove that the dual of a poset is also a poset. **(4+4+8)**

8. a) If $\{L, \leq\}$ is a lattice in which $\vee$ and $\wedge$ denote the operations of join and meet respectively, then for a, b, $C \in L$ prove the following

i. Isotonic property

ii. Distributive Inequalities.

iii. Modular inequalities.

b) In a distributive lattice $(L, \vee, \wedge)$ if an element $a \in L$ has a complement, then prove that it is unique. **(10+6)**
