MMDSE-3.4

Sl.No. : **Total No. of Pages : 2**

III Semester M.Sc. Degree Examination, February - 2024
(SLM Scheme CBCS)
MATHEMATICS
Course-IV : Number Theory

Time : 3 Hours **Max. Marks : 80**

Instruction : ***Answer any five full questions.***

Q1) a) Define Greatest common divisor If a and b are any two integers, not both of them are zero then prove that there exists integer x and y such that gcd $(a,b) = ax + by.$

b) State and prove prime number theorem.

[8+8]

Q2) a) Prove that if $ca \equiv cb \pmod{n}$ then $a \equiv b \pmod{n/d}$, where d = scd (c,n).

b) State and prove wilson's theorem.

c) If $n \geq 1$ and Gcd $(a,n) = 1$ then P.T $a^{\phi(n)} \equiv 1 (\text{mod} n)$.

[5+5+6]

Q3) a) If $n = P_1^{k_1} P_2^{k_2} ... P_r^{k_r}$ is the prime factroisation of $n > 1$ then prove the followings.

i) $\tau(n) = (k_1 + 1)(k_2 + 1) ..... (k_r + 1)$

ii) $\sigma(n) = \frac{p_1^{k_1+1}}{p_1 - 1} \frac{p_2^{k_2+1}}{p_2 - 1} ..... \frac{p_r^{k_r+1} - 1}{P_r - 1}$

b) For any positive integer n then P.T

$$\phi(n) = n \sum_{d/n} \frac{M(d)}{d}$$

[8+8]

Q4) a) Using RSA enciphering key $(e,n)=(11,2867)$ encrypt each message SEAFOOD and OPEN DOOR.

b) Show that if f and g all Multiplicative so is their Dirichlet product $f*g$.

c) Per $x>1$, then prove that $\sum_{n\le x}\phi(n)=\frac{3}{\pi^2}x^2+0\,(x\log x)$

so the average order of $\phi(n)$ is $\frac{3n}{\pi^2}$.

[5+6+5]

Q5) a) If p is a prime number and d/p-1 then show that there are exactly $\phi(d)$ incongruent integers having order d mode.

b) If $m,n>2$ and gcd $(m,n)=1$ then show that there exists no primitive root (modmn).

c) Solue the congruence of $x^3\equiv 5\,(\text{mod } 13)$.

[5+6+5]

Q6) a) State and prove Gauss lemma.

b) Evaluate of $\left(-23/59\right)$

c) Solve the congruence $x^2\equiv 91\,(\text{mod } 27)$

[7+4+5]

Q7) a) Show that if an intiser n can be expressed as the sum of two squares in two different ways, then n is composite.

b) P.T $F_n^2=F_{n+1}F_{n-1}+(-1)^{n-1}$

c) Express 459 as sum of four squares

[6+5+5]

Q8) a) Define periodic confined fractions If the number x has a periodic simple continued fraction expansion them prove that x is a quadratic irrational.

b) Write down $x_0=\sqrt{15}$ irrational number as an infinite Simple Continued fraction.

c) Explain pell's equation.

[6+5+5]

