MMDSE-3.4

Sl.No. :

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III Semester M.Sc. Degree Examination, June/July-2024
(CBCS, SLM Scheme)
MATHEMATICS
Course-IV : Number Theory

Time : 3 Hours **Max. Marks : 80**

Instruction : Answer any five full questions.

Q1) a) State and prove fundamental theorem of arithmetic.

b) Show that there is an infinite number of primes.

c) Prove that the linear Diophantive equation $ax + by = c$ has a solution if and only if d/c where d = gcd (a, b). If (x_0, y_0) is any particular solution, then all other solutions are given by $x = x_0 - \left(\frac{b}{d}\right) t$ and $y = y_0 + \left(\frac{a}{d}\right) t$.

Where t is any arbitary integer.

[6+5+5]

Q2) a) State and prove Fermat's little theorem.

b) If p is a prime and k > 0 then P. T

$$\phi\left(p^K\right) = p^K - p^{K-1} = p^K\left(1 - \frac{1}{p}\right)$$

c) For $n > 2$ then prove $\phi(n)$ is an even integer.

[6+6+4]

Q3) a) Show that the functions τ and σ are both multiplicative functions.

b) If $n = p_1^{k_1} p_2^{k_2} \ldots\ldots p_r^{k_r}$ is the prime factorization of $n > 1$ then prove the positive divisors of n are preciscly those intigers d of the form

$d = p_1^{a_1}, p_2^{a_2} \ldots\ldots p_r^{a_r}$, where $O \leq a_i \leq k_i (i = 1, 2, \ldots\ldots, r)$.

c) Show that for each positive integer $n \geq 1$ $n = \sum_{d/n} \phi(d)$ the sum being extended over all positive divisors of n.

[6+5+5]

Q4) a) Using the ceaser cipher, encipher the following "ALL IS WELL THAT ENDS WELL".

b) Prove that Dirichlet multiplication is commutative and associative.

c) For all $x \geq 1$ then show that

$$\sum_{n \leq x} d\ (n) = x \log x + (2C - 1)\ x + O\ (\sqrt{x}).$$

Where C is Euler's constant.

[5+5+6]

Q5) a) State and prove Lagrange's theorem.

b) Show that integers of the form 2^n has no primitive root for $n \geq 3$.

c) Construct the index (mod 18)

[6+5+5]

Q6) a) State and prove Euler's criterion.

b) Evaluate n of Gauss Lemma for $\left(\frac{11}{23}\right)$.

c) If p is an odd prime and a is any integer such that $(a,p) = 1$ then prove that the congruence $x^2 = a \pmod{p^n}$, $n \geq 1$ has a solution if and only if $\left(\frac{a}{p}\right) = 1$.

[7+3+6]

Q7) a) Prove that an odd prime p is expressible as a sum of two squares iff $p \equiv 1 \pmod 4$.

b) Show that any positive integer n can be expressed as a sum of four squares.

c) P. T $F_{m+n} = F_{m-1} F_n + F_m F_{n+1}$

[6+6+4]

Q8) a) Define finite continued fractions. Prove that every finite simple continued fraction is equal to a rational number and every rational number can be written as a finite simple continued fraction.

b) Explain Pell's equation.

c) Write down $x_0 = \sqrt{23}$ irrational number as an infinite simple continued fraction.

[6+5+5]

