# MMDSE-3.4

Sl.No. : **Total No. of Pages : 3**

## III Semester M.Sc. Degree Examination, May/June - 2026

## (SLM Scheme, CBCS)

## MATHEMATICS

## Course - IV : Number Theory

**Time : 3 Hours** **Max. Marks : 80**

Answer any five questions.

***Q1)*** a) If $P_n$ is the $n^{th}$ prime, prove that $P_n \leq 2^{2n-1}$.

b) Is 509 a prime number and find six consecutive integers that are composite.

c) Prove that there is no Polynomial $f(n)$ with integral Co-efficients that will produce Primes for all integers n.

d) Prove there is no end to the sequence of primes 2, 3, 5, 7,11,13.....

**[4+4+4+4]**

***Q2)*** a) Let $f_n$ denote the $n^{th}$ Fermat number. Prove that $f_n = f_{n-1}^2 - 2f_{n-1} + 2$, When $n \geq 1$. Also show that $641 / f_5$.

b) Find the solution of the system of congruences: $3x + 4y \equiv 5 (\text{mod} 13), 2x + 5y \equiv 7 (\text{mod} 13)$.

c) Prove that $\pi^2$ is irrational.

**[6+5+5]**

***Q3)*** a) Define Dirichlet product of arithmetical function. Let $f$ be an arithmetical function with $f(1) \neq 0$, then prove that there is a unique function g such that $f$*g = I.

b) If both g and f*g are multiplicative, Prove that $f$ is also multiplicative.

c) If d/n and k∈N. Show that $\phi(nd^k) = d^k \phi(n)$.

d) Find the largest prime number P such that $P^2 / \phi(95!)$

**[4+4+4+4]**

***Q4)*** a) Encipher each using affine cipher $C \equiv 3p + 7 (\text{mod}\, 26)$; A THING OF BEAUTY IS JOY FOR EVER.

b) Prove that for all $x \geq 1, \sum_{n \leq x} d(n) = x \log x + (2r-1)x + 0\left(\sqrt{x}\right)$.

c) For $x > 1$, then prove that $\sum_{n \leq x} \phi(n) = \frac{3}{\pi^2} x^2 + 0(x \log x)$ so the average order of $\phi(n)$ is $\frac{3n}{\pi^2}$.

**[6+5+5]**

***Q5)*** a) If P is a prime number and d/p–1 then show that there are exactly $\phi(d)$ in congruent integers having order d mode.

b) If *a* is *a* primitive root modulom, then prove that:

i) $\{1, a, a^2, ...., a^{\phi(m)-1}\}$ is a reduced residue system modulo m.

ii) $a^k$ is also a primitive root modulo M if K and $\phi$(m) are relatively prime.

**[8+8]**

***Q6)*** a) Solve the Congruence $17x^{20} \equiv 19 (\text{mod}\, 37)$.

b) Let p be an odd prime and let a, b$\in$z such that gcd (a,b) = 1. Then prove that:

i) $\left(\frac{a}{p}\right) = a^{\left(\frac{p-1}{2}\right)} (\text{mod}\, p)$

ii) $\left(\frac{a}{p}\right)\left(\frac{b}{p}\right) = \left(\frac{ab}{p}\right)$

iii) a ≡ b(modp) implies $\left(\frac{a}{p}\right) = \left(\frac{b}{p}\right)$

iv) $\left(\frac{a^2}{p}\right) = 1$

c) Evaluate $\left(\frac{-23}{59}\right)$

**[4+8+4]**

***Q7)*** a) Prove that any positive integer n can be written as sum of four squares, some of which may be zeros express 459 as the Sum of four squares.

b) Prove that for a prime $p > 5$, either $p/u_{p-1}$ or $P/u_{p+1}$, but not both, where $u_n$ is a Fibonacci number

**[8+8]**

***Q8)*** a) Show that any simple continued fraction is periodic if and only if it is quadratic irrational number.

b) Determine a Particular solution of Diophantine equation $43x + 5y = 1$ by means of continued fraction.

**[14+2]**

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